Collective behaviour

fuzzy sets
fuzzy arithmetics
fuzzy logic

mehka logika?

regulacija sobne temperature

if the temperature is *just right*
leave as is;

if the temperature is *too cold*
increase heating;

if the temperature is *too hot*
decrease heating;

MNOŽICE

Zapis z naštevanjem
$\mathbf{A} = \{1,2,3,4,5\};\ \mathbf{A} = [2,4]$

Zapis z lastnostjo
$\mathbf{A} = \{x \in \mathbb{R} |\ 2 \le x \le 4\}$

Pripadnostna funkcija
$\mu_{\mathbf{A}}: \mathbf{X} \to \{0,1\}$

$$\mu_{\mathbf{A}}(x) = \begin{cases} 1 & x \in \mathbf{A} \\ 0 & x \notin \mathbf{A} \end{cases}$$

$$\mu_{\mathbf{A}}(x) = \begin{cases} 1 & \text{iff } x \in [2,4] \\ 0 & \text{otherwise} \end{cases}$$

Dobro definirani koncepti

$\mathbf{short} = \{x \in \mathbb{R}; x < 180\}$
$\mathbf{tall} = \{x \in \mathbb{R}; x \ge 180\}$

mehka logika

regulacija sobne temperature

if the temperature is *just right*
leave as is;

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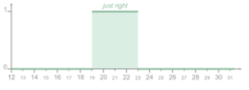
increase heating;

if the temperature is *too hot*
decrease heating;



$$just\ right = \{x \in \mathbb{R}; 19 \leq x \leq 23\}$$

mehka logika

regulacija sobne temperature

if the temperature is *just right*
leave as is;

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increase heating;

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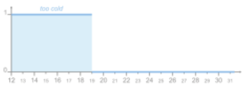
decrease heating;



$$too\ cold = \{x \in \mathbb{R}; x < 19\}$$

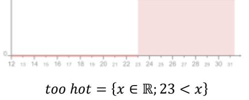
mehka logika

regulacija sobne temperature

if the temperature is *just right*
leave as is;

if the temperature is *too cold*
increase heating;

if the temperature is *too hot*
decrease heating;



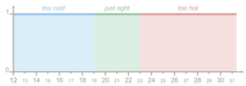
$$too\ hot = \{x \in \mathbb{R}; 23 < x\}$$

mehka logika

regulacija sobne temperature

```
if (temperature > 12)&&(temperature < 19)
    then (heating += 1),

if (temperature > 23)&&(temperature < 32)
    then (heating -= 1).
```

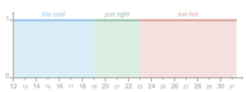


mehka logika

regulacija sobne temperature

```
if (temperature is too cold)
    then (heating += 1),

if (temperature is too hot)
    then (heating -= 1).
```



MEHKE MNOŽICE

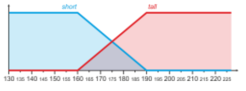
Pripadnostna funkcija

$$\mu_F: X \to [0,1]$$

$$\mu_F(x) = \begin{cases} x-3 & \text{iff } x \in [2,3] \\ 4-x & \text{iff } x \in [3,4] \\ 0 & \text{otherwise} \end{cases}$$

Dvoumni/nejasni koncepti

full, empty, cold, hot, …

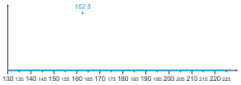


$$\mu_{\mathbf{short}}(x) = \begin{cases} 1 & \text{iff } x < 160 \\ (190-x)/30 & \text{iff } x \in [160,190] \\ 0 & \text{otherwise} \end{cases}$$

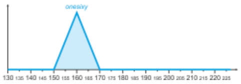
mehke množice

singletone
trikotna
trapezoidna
gausova

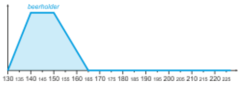
$$S[m];\ \mu_F(x) = \begin{cases} 1 & x = m \\ 0 & \text{othterwise} \end{cases}$$



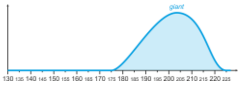
$$TRI[l,m,r];\ \mu_F(x) = \begin{cases} \frac{x-l}{m-l} & x \le m \\ \frac{r-x}{r-m} & x > m \\ 0 & \text{othterwise} \end{cases}$$



$$TRAP[l,ml,mr,r];\ \mu_F(x) = \begin{cases} \frac{x-l}{ml-l} & x < ml \\ \frac{r-x}{r-mr} & x > mr \\ 1 & ml \le x \le mr \\ 0 & \text{othterwise} \end{cases}$$



$$G[\sigma_l,m,\sigma_r];\ \mu_F(x) = \begin{cases} e^{-\frac{(x-m)^2}{2\sigma_l^2}} & x \le m \\ e^{-\frac{(x-m)^2}{2\sigma_r^2}} & x > m \end{cases}$$



MNOŽICE

Komplement

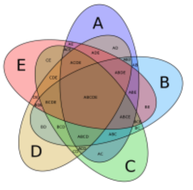
$$C = \overline{A};\ \mu_C(x) = \begin{cases} 1 & \text{iff } x \notin A \\ 0 & \text{otherwise} \end{cases}$$

Presek

$$C = A \cap B;\ \mu_C(x) = \begin{cases} 1 & \text{iff } x \in A \textbf{ and } x \in B \\ 0 & \text{otherwise} \end{cases}$$

Unija

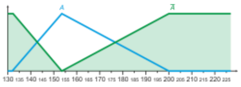
$$C = A \cup B;\ \mu_C(x) = \begin{cases} 1 & \text{iff } x \in A \textbf{ or } x \in B \\ 0 & \text{otherwise} \end{cases}$$



Komplement

MEHKE MNOŽICE

$$C = \overline{A};\ \mu_C(x) = 1 - \mu_A(x)$$



Presek

MEHKE MNOŽICE

Minimum

$C = A \cap B; \quad \mu_C(x) = \min(\mu_A(x), \mu_B(x))$

Produkt

$C = A \cap B; \quad \mu_C(x) = \mu_A(x)\mu_B(x)$

t-norma ⋆

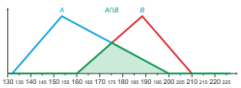
$C = A \cap B; \quad \mu_C(x) = \mu_A(x) \star \mu_B(x)$

Omejeni produkt

$a \star b = \max(0, a + b - 1)$

Drastični produkt

$$a \star b = \begin{cases} a & \text{iff } b = 1 \\ b & \text{iff } a = 1 \\ 0 & \text{otherwise} \end{cases}$$



Unija

MEHKE MNOŽICE

Maximum

$C = A \cup B; \quad \mu_C(x) = \max(\mu_A(x), \mu_B(x))$

Vsota

$C = A \cup B; \quad \mu_C(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$

s-norma ⋄

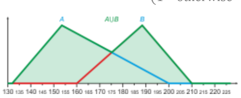
$C = A \cup B; \quad \mu_C(x) = \mu_A(x) \diamond \mu_B(x)$

Omejena vsota

$a \diamond b = \min(1, a + b)$

Drastična vsota

$$a \diamond b = \begin{cases} a & \text{iff } b = 0 \\ b & \text{iff } a = 0 \\ 1 & \text{otherwise} \end{cases}$$



MNOŽICE

Komutativnost

$A \cap B = B \cap A, \quad A \cup B = B \cup A$

Asociativnost

$A \cap (B \cap C) = (A \cap B) \cap C, \quad A \cup (B \cup C) = (A \cup B) \cup C$

Distributivnost

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Idempotenca

$A \cap A = A, \quad A \cup A = A$

Absorpcija

$A \cap (A \cup B) = A, \quad A \cup (A \cap B) = A, \quad A \cap \emptyset = \emptyset, \quad A \cup \mathbb{U} = \mathbb{U}$

Identieta

$A \cap \mathbb{U} = A, \quad A \cup \emptyset = A$

De Morganovo pravilo

$\overline{A \cap B} = \overline{A} \cup \overline{B}, \quad \overline{A \cup B} = \overline{A} \cap \overline{B}$

Pravilo izključitve tretjega

$A \cup \overline{A} = \mathbb{U}$

Pravilo kontradiktornosti

$A \cap \overline{A} = \emptyset$

MEHKA ŠTEVILA

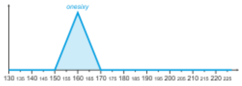
Pripadnostna funkcija μ_{A} mora biti

normalna, $\exists x: \mu_{\mathrm{A}}(x) = 1$

konveksna

odsekoma zvezna

$$\mu_{\mathrm{A}}(x) = \langle \overline{x}, \alpha, \beta \rangle_{LR} = \begin{cases} L[(\overline{x} - x)/\alpha] & x < \overline{x} \\ R[(x - \overline{x})/\beta] & x > \overline{x} \end{cases}$$



mehka aritmetika

seštevanje

odštevanje

množenje

deljenje

$$\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} + \langle \overline{x}_2, \alpha_2, \beta_2 \rangle_{LR} = \langle \overline{x}_1 + \overline{x}_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2 \rangle_{LR}$$

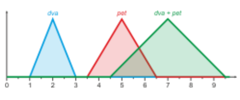
$$-\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} = \langle -\overline{x}_1, \alpha_1, \beta_1 \rangle_{LR}$$

$$\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} - \langle \overline{x}_2, \alpha_2, \beta_2 \rangle_{LR} = \langle \overline{x}_1 - \overline{x}_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2 \rangle_{LR}$$

$$\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} \times \langle \overline{x}_2, \alpha_2, \beta_2 \rangle_{LR} \approx \langle \overline{x}_1 \overline{x}_2, \overline{x}_1 \alpha_2 + \overline{x}_2 \alpha_1, \overline{x}_1 \beta_2 + \overline{x}_2 \beta_1 \rangle_{LR}$$

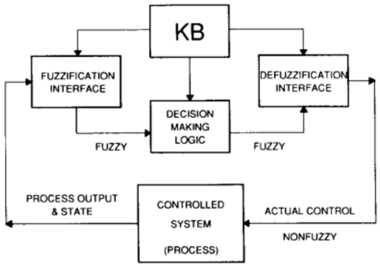
$$\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} \times \langle \overline{x}_2, \alpha_2, \beta_2 \rangle_{LR} \approx \langle \overline{x}_1 \overline{x}_2, \overline{x}_1 \alpha_2 + \overline{x}_2 \alpha_1 - \alpha_1 \alpha_2, \overline{x}_1 \beta_2 + \overline{x}_2 \beta_1 - \beta_1 \beta_2 \rangle_{LR}$$

$$\langle \overline{x}_1, \alpha_1, \beta_1 \rangle_{LR} \div \langle \overline{x}_2, \alpha_2, \beta_2 \rangle_{LR} \approx \left\langle \overline{x}_1 / \overline{x}_2, (\overline{x}_1 \alpha_2 + \overline{x}_2 \alpha_1)/\overline{x}_2^2, (\overline{x}_1 \beta_2 + \overline{x}_2 \beta_1)/\overline{x}_2^2 \right\rangle_{LR}$$



Mehki krmilnik

FLC



LOGIKA

Množice

$\subseteq, \quad \not\subseteq, \quad \overline{}, \quad \cap, \quad \cup, \quad \equiv$

Logika

$T, \quad F, \quad \neg, \quad \wedge, \quad \vee, \quad \equiv$

Implikacija

$\textbf{if } p \textbf{ then } q, \quad p \rightarrow q \equiv \neg p \vee q$

Modus Ponens

$\textbf{if } (x \textbf{ is } \mathrm{A}) \textbf{ then } (y \textbf{ is } \mathrm{B})$ – *implikacija*
$(x \textbf{ is } \mathrm{A})$ – *predpostavka*
$(y \textbf{ is } \mathrm{B})$ – *sklep*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

MEHKA LOGIKA

Posplošeni Modus Ponens

$\textbf{if } (x \textbf{ is } \mathrm{A}) \textbf{ then } (y \textbf{ is } \mathrm{B})$ – *implikacija*
$(x \textbf{ is } \mathrm{A}^*)$ – *predpostavka*
$(y \textbf{ is } \mathrm{B}^*)$ – *sklep*

$$\mu_{\mathrm{B}^*}(y) = \sup_x [\mu_{\mathrm{A}^*}(x) \star \mu_{\mathrm{A}\rightarrow\mathrm{B}}(x, y)]$$

Mamdani implikacija

$$\mu_{\mathrm{A}\rightarrow\mathrm{B}}(x, y) = \mu_{\mathrm{A}}(x) \star \mu_{\mathrm{B}}(y)$$

Minimum

$a \star b = \min(a, b)$

Produkt

$a \star b = ab$

$\mu_{\mathrm{A}}(x)$	$\mu_{\mathrm{B}}(y)$	$\min(\mu_{\mathrm{A}}(x), \mu_{\mathrm{B}}(y))$	$\mu_{\mathrm{A}}(x)\mu_{\mathrm{B}}(y)$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

Mamdani implikacija

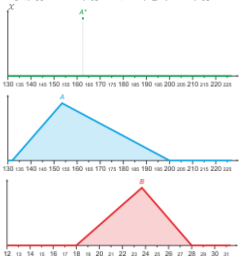
MEHKA LOGIKA

$$\mu_{\mathrm{B}^*}(y) = \sup_x [\mu_{\mathrm{A}^*}(x) \star (\mu_{\mathrm{A}}(x) \star \mu_{\mathrm{B}}(y))] = \mu_{\mathrm{B}}(y) \star \sup_x [\mu_{\mathrm{A}^*}(x) \star \mu_{\mathrm{A}}(x)]$$

Singletone A^*

$$\mu_{\mathrm{A}^*}(x) = \begin{cases} 1 & x = x' \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\mathrm{B}^*}(y) = \mu_{\mathrm{B}}(y) \star \sup_x [\mu_{\mathrm{A}^*}(x) \star \mu_{\mathrm{A}}(x)] = \mu_{\mathrm{B}}(y) \star \mu_{\mathrm{A}}(x')$$



$$\mu_{B^*}(y) = \sup_x\left[\mu_{A^*}(x) \star \left(\mu_A(x) \star \mu_B(y)\right)\right] = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right]$$

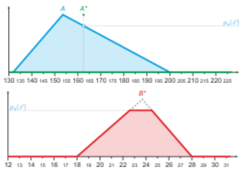
Singletone A*

$$\mu_{A^*}(x) = \begin{cases} 1 & x = x' \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{B^*}(y) = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right] = \mu_B(y) \star \mu_A(x')$$

Mamdani implikacija

MEHKA LOGIKA



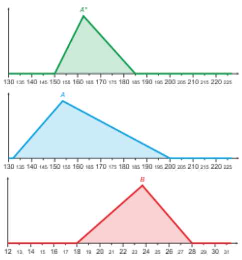
$$\mu_{B^*}(y) = \sup_x\left[\mu_{A^*}(x) \star \left(\mu_A(x) \star \mu_B(y)\right)\right] = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right]$$

Non-Singletone A*

$$\mu_{B^*}(y) = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right]$$

Mamdani implikacija

MEHKA LOGIKA



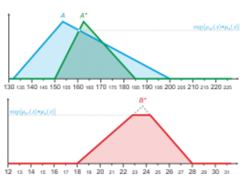
$$\mu_{B^*}(y) = \sup_x\left[\mu_{A^*}(x) \star \left(\mu_A(x) \star \mu_B(y)\right)\right] = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right]$$

Non-Singletone A*

$$\mu_{B^*}(y) = \mu_B(y) \star \sup_x\left[\mu_{A^*}(x) \star \mu_A(x)\right]$$

Mamdani implikacija

MEHKA LOGIKA



mehka logika

več pogojev

več pogojnih stavkov

Konjunkcija

if $(a$ **is A**$)$ **and** $(b$ **is B**$)$ **then** $(c$ **is C**$)$

$$\mu_{\mathbf{C}^*}(z) = \mu_{\mathbf{C}}(z) \star \left(\sup_x[\mu_{\mathbf{A}^*}(x) \star \mu_{\mathbf{A}}(x)] \star \sup_y[\mu_{\mathbf{B}^*}(y) \star \mu_{\mathbf{B}}(y)]\right)$$

$$\mu_{\mathbf{C}^*}(z) = \min\left(\mu_{\mathbf{C}}(z), \min\left(\mu_{\mathbf{A}}(x'), \mu_{\mathbf{B}}(y')\right)\right)$$

Disjunkcija

if $(a$ **is A**$)$ **or** $(b$ **is B**$)$ **then** $(c$ **is C**$)$

$$\mu_{\mathbf{C}^*}(z) = \mu_{\mathbf{C}}(z) \star \left(\sup_x[\mu_{\mathbf{A}^*}(x) \star \mu_{\mathbf{A}}(x)] \diamond \sup_y[\mu_{\mathbf{B}^*}(y) \star \mu_{\mathbf{B}}(y)]\right)$$

$$\mu_{\mathbf{C}^*}(z) = \min\left(\mu_{\mathbf{C}}(z), \max\left(\mu_{\mathbf{A}}(x'), \mu_{\mathbf{B}}(y')\right)\right)$$

Agregacija

if $(a$ **is A**$)$ **then** $(c$ **is D**$)$

if $(b$ **is B**$)$ **then** $(c$ **is E**$)$

$$\mu_{\mathbf{D}^*}(z) = \mu_{\mathbf{D}}(z) \star \sup_x[\mu_{\mathbf{A}^*}(x) \star \mu_{\mathbf{A}}(x)]$$

$$\mu_{\mathbf{E}^*}(z) = \mu_{\mathbf{E}}(z) \star \sup_y[\mu_{\mathbf{B}^*}(y) \star \mu_{\mathbf{B}}(y)]$$

$$\mathbf{C}^* = \mathbf{D}^* \cup \mathbf{E}^*$$

$$\mu_{\mathbf{C}^*}(z) = \mu_{\mathbf{D}^*}(z) \diamond \mu_{\mathbf{E}^*}(z) = \max\left(\min\left(\mu_{\mathbf{D}}(z), \mu_{\mathbf{A}}(x')\right), \min\left(\mu_{\mathbf{E}}(z), \mu_{\mathbf{B}}(y')\right)\right)$$

Defuzifikacija

MEHKA LOGIKA

Centroid

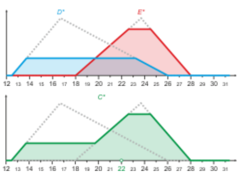
$$\mathbf{C}^* = \mathbf{D}^* \cup \mathbf{E}^*$$

$$y = \frac{\sum_i y_i \, \mu_{\mathbf{C}^*}(y_i)}{\sum_i \mu_{\mathbf{C}^*}(y_i)}$$

Center-of-sums

$$\mu_{\mathbf{C}^*}(y) = \mu_{\mathbf{D}^*}(y) + \mu_{\mathbf{E}^*}(y)$$

Height defuzifier



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mehka logika

naravni jezik

Linguistic hedge

$$\mu_{\mathbf{A}^*}(x) = \mu_{\mathbf{A}}(x)^{\alpha}$$

$\alpha = 2$ … “very” (slo. *zelo*)

$\alpha = 4$ … “very, very” (slo. *zelo*, *zelo*)

$\alpha = 1.25$ … “plus” (slo. *več*)

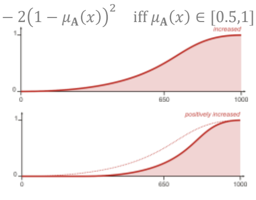
$\alpha = 0.5$ … “slightly” (slo. *rahlo*)

$\alpha = 0.75$ … “minus” (slo. *manj*)

…

Zadeh (1972) “intensify”

$$\mu_{\mathbf{A}^*}(x) = \begin{cases} 2\mu_{\mathbf{A}}(x) & \text{iff } \mu_{\mathbf{A}}(x) \in [0{,}0.5] \\ 1 - 2\left(1 - \mu_{\mathbf{A}}(x)\right)^2 & \text{iff } \mu_{\mathbf{A}}(x) \in [0.5{,}1] \end{cases}$$



Brain divided

© J Haworth, et al., goo.gl/l6y2Ql, vimeo.com/70308089

mehka logika

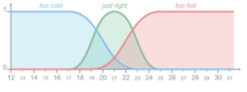
regulacija sobne temperature

mehke množice

```
if (temperature is just right)
   then (heating is as is),

if (temperature is too cold)
   then (heating is increase),

if (temperature is too hot)
   then (heating is decrease).
```



mehka logika

regulacija sobne temperature

mehke množice

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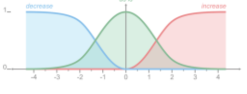
if (temperature is too hot)
   then (heating is decrease).
```



mehka logika

regulacija sobne temperature

- mehke množice
- mehko sklepanje

```
if (temperature is just right)
   then (heating is as is),

if (temperature is too cold)
   then (heating is increase),

if (temperature is too hot)
   then (heating is decrease).
```

```
temperature = 18.7
```

mehka logika

regulacija sobne temperature

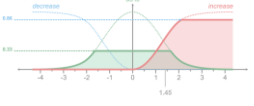
- mehke množice
- mehko sklepanje

```
if (temperature is just right)
   then (heating is as is),

if (temperature is too cold)
   then (heating is increase),

if (temperature is too hot)
   then (heating is decrease).
```

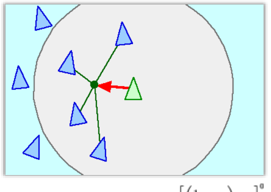
```
temperature = 18.7
   => heating += 1.45
```



jate ptic

modeliranje in simulacija

- težnja bližine



$$\mathbf{F}_A = \left[\left(\frac{1}{n}\sum_i \mathbf{p}_i\right) - \mathbf{p}\right]^0$$

jate ptic

modeliranje in simulacija
težnja bližine

v splošnem *ohrani* smer in hitrost leta;

če je sosed *dovolj blizu*,
ohrani smer in hitrost leta;

če je sosed *predaleč* in *spredaj*,
pospeši let;

če je sosed *predaleč* in kjerkoli *levo ali zadaj*,
se *usmeri proti njemu* in *upočasni* let;

če je sosed *predaleč* in kjerkoli *desno ali zadaj*,
se *usmeri proti njemu* in *upočasni* let.

jate ptic

modeliranje in simulacija
težnja bližine

```
if (distance is too far)
   then (flight speed is keep speed),

if (distance is too far)
   then (flight direction is keep direction),

if (distance is close enough)
   then (flight speed is keep speed),

if (distance is close enough)
   then (flight direction is keep direction),

if (distance is too far) and (position is in front)
   then (flight speed is accelerate),

if (distance is too far) and (position is left or behind)
   then (flight speed is decelerate),

if (distance is too far) and (position is left or behind)
   then (flight direction is turn left),

if (distance is too far) and (position is right or behind)
   then (flight speed is decelerate),

if (distance is too far) and (position is right or behind)
   then (flight direction is turn right).
```

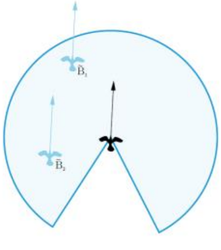


Figure 5.5 The observed fuzzy animat perceives two neighbours, one of which is 80% of the visual range away with an angular offset of -30° ($\tilde{B}_1$), and the other is 60% of the visual range away with an angular offset of -110° ($\tilde{B}_2$). All fuzzy animats have the same flight direction and flight speed (black and blue arrows).

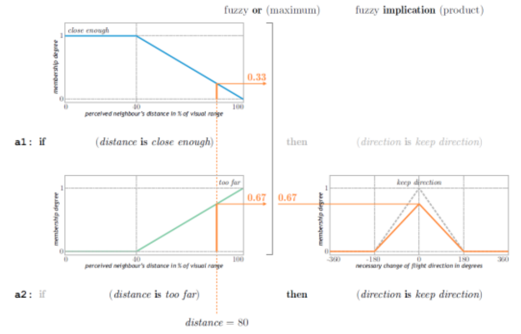


Figure 5.6 Graphical representation of the evaluation of rules a1 and a2 for the case when the perceived neighbour is 80% of the visual range away with an angular offset of $-30°$. The left half shows the evaluation of the degrees of truth of the antecedents and the right part the modification of the output fuzzy set.

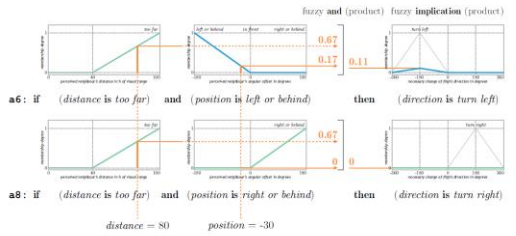


Figure 5.7 Graphical representation of the evaluation of rules a6 and a8 for the case when the perceived neighbour is 80% of the visual range away with an angular offset of $-30°$. The left half shows the evaluation of the degrees of truth of the antecedents and the right part the modification of the output fuzzy sets.

jate ptic

modeliranje in simulacija

težnja bližine

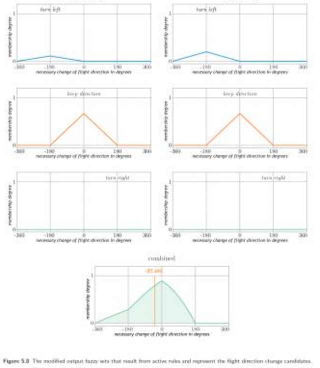
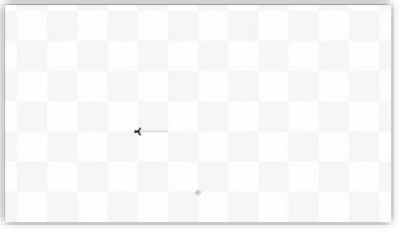


Figure 5.8 The modified output fuzzy sets that result from active rules and represent the flight direction change candidates. They were obtained by evaluating the attraction drive fuzzy rule base for the case when the only two perceived neighbours are 80% of the visual range away with an angular offset of $-30°$ ($\hat{B}_1$), and 60% of the visual range away with an angular offset of $-110°$ ($\hat{B}_2$). The combined fuzzy set is the algebraic sum fuzzy union of the modified output fuzzy sets. The defuzzified value, obtained by using the centroid method, equals $-37.485°$.

jate ptic

modeliranje in simulacija

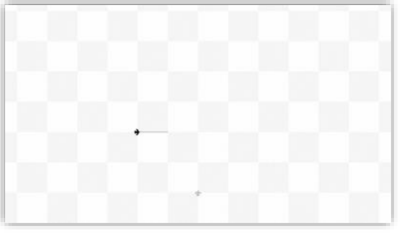
težnja bližine



jate ptic

modeliranje in simulacija

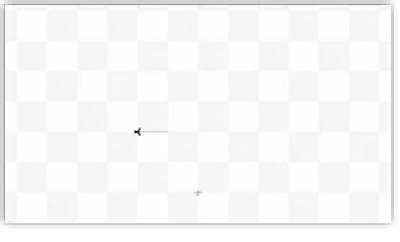
težnja razdalje



jate ptic

modeliranje in simulacija

težnja usklajenosti



jate ptic

modeliranje in simulacija