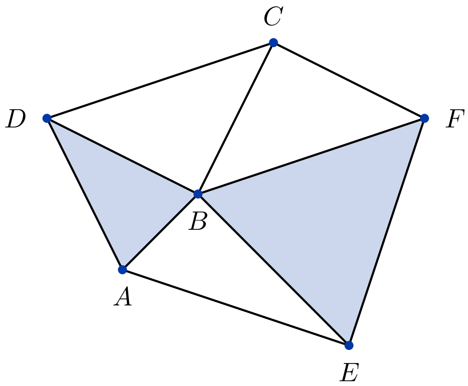
Computational topology

Lab work, 3rd week

1. Let $X_n = S^n \setminus \{(0,\dots,0,1),(0,\dots,0,-1)\} \subset \mathbb{R}^{n+1}$ and $Y_n = S^{n-1} \times (-1,1) \subset \mathbb{R}^{n+1}$. Draw X_n and Y_n for $n = 0,1,2$. Prove that X_2 and Y_2 are homeomorphic.

2. Find the open stars $\mathrm{st}(A)$, $\mathrm{st}(AB)$ and the links $\mathrm{lk}(A)$, $\mathrm{lk}(AB)$ for the simplicial complex given below.



3. The simplicial complex K contains the following simplices:

$$\langle v_0\rangle, \langle v_1\rangle, \langle v_2\rangle, \langle v_3\rangle, \langle v_4\rangle, \langle v_0, v_1\rangle, \langle v_0, v_3\rangle, \langle v_1, v_3\rangle, \langle v_0, v_1, v_2\rangle.$$

   (a) Add any simplices that are missing from K.

   (b) Draw the Hasse diagram of K.

   (c) Draw the open stars $\mathrm{st}(\langle v_0\rangle)$, $\mathrm{st}(\langle v_0, v_1\rangle)$ and the links $\mathrm{lk}(\langle v_0\rangle)$, $\mathrm{lk}(\langle v_0, v_1\rangle)$. Mark them on the Hasse diagram as well.

4. For each of the following triangulations determine if it is a triangulation of a surface.

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A: [(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)]
B: [(1, 2, 3), (1, 2, 4), (2, 3, 5), (2, 3, 6), (3, 5, 7)]
C: [(1, 2, 3), (2, 3, 4), (3, 4, 5),
    (4, 5, 6), (1, 5, 6), (1, 2, 6)]
D: [(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),
    (1, 4, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),
    (4, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),
    (2, 3, 9), (3, 5, 9), (1, 3, 5)]
E: [(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),
    (1, 5, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),
    (5, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),
    (2, 3, 9), (3, 4, 9), (1, 3, 4)]
F: [(1, 2, 3), (1, 3, 4), (2, 3, 4), (4, 5, 6)]
G: [(1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (2, 5, 6), (1, 2, 6)]
H: [(1, 3, 5), (1, 2, 6), (1, 5, 6), (1, 2, 4), (1, 3, 4),
    (2, 3, 5), (2, 3, 6), (2, 4, 5), (3, 4, 6), (4, 5, 6)]
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(a) Find the Euler characteristics for all of these simplicial complexes.

(b) For each case check if the given triangulation belongs to a surface (a 2-dimensional triangulated manifold).

(c) Find the number of boundary components for all of the surfaces.

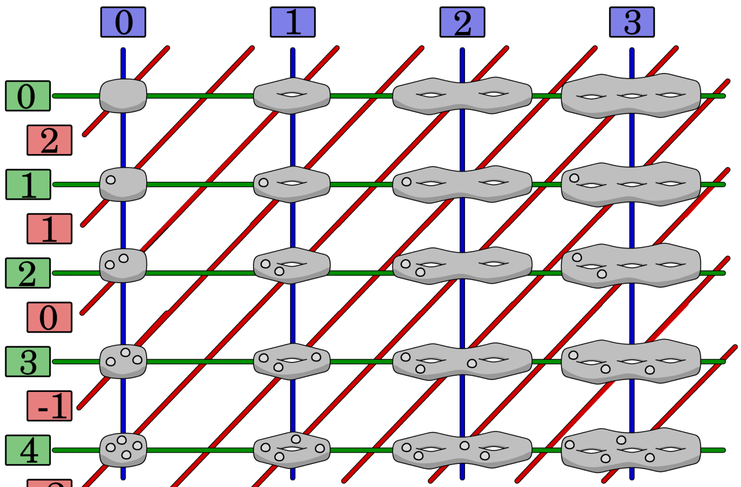
(d) For each of the surfaces determine if it is orientable or not.

(e) Determine the genus of each orientable surface and the genus of non-orientable surfaces with no boundary.

(f) Name each of the surfaces.

Use the following array to keep track of the results.

	Euler characteristic	manifold Y/N	# of boundary components	orientable Y/N	genus	name
A						
B						
C						
D						
E						
F						
G						
H						



$$2 = 2g + \chi + \#\partial$$

genus	0	1	2
orientable	S^2	T	$T\#T$
non-orientable		P	$P\#P$