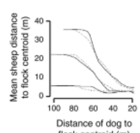
Collective behaviour

sheep

King et al. 2012 doi: 10.1016/j.cub.2012.05.008

$$\dot{x} = \begin{cases} 0 & d \geq d_0 \\ -s\left(\frac{2}{1 + \exp[-(x - x_0)/\lambda]} - 1\right) & d < d_0 \end{cases}$$

x... distance of sheep to centre of group
d... distance of shepherd to centre of group
d_0, x_0, s, λ... parameters ($x_0 = 4.2$, $d_0 = 73.7$, $s = 34.5$, and $\lambda = 0.41$)



Strömbom et al. 2014 doi: 10.1098/rsif.2014.0719

Universe: $L \times L$ field, N sheep ($\mathbf{A}_i, i = 1, \dots, N$) and 1 shepherd ($\mathbf{S}$)

Initial state: sheep placed randomly at the right-hand quadrant of the field

Shepherd task: herd sheep to target location in lower-left quadrant

$$\mathbf{A}_i = \mathbf{A}_i + \delta\widehat{\mathbf{H}}_i$$
$$\mathbf{H}_i' = h\widehat{\mathbf{H}}_i + c\widehat{\mathbf{C}}_i + \rho_a\widehat{\mathbf{R}}_i^a + \rho_s\widehat{\mathbf{R}}_i^s + e\widehat{\boldsymbol{\epsilon}}_i$$
$$\rho_a > c > \rho_s > h$$

Sheep behaviour

- Keep at safe distance from nearest neighbours ($\mathbf{R}_i^a = \sum_{\mathbf{A}_j \in \mathbf{N}} \frac{\mathbf{A}_i - \mathbf{A}_j}{\|\mathbf{A}_i - \mathbf{A}_j\|}, N = \{\mathbf{A}_j : \|\mathbf{A}_j - \mathbf{A}_j\| < r_a\}$
- If shepherd not visible ($\|\boldsymbol{S} - \boldsymbol{A}_i\| > r_s$) remain stationary, but exhibit small random movements (if $\xi < p, \boldsymbol{\epsilon} = (\cos\zeta, \sin\zeta)$, if $\xi \geq p, \boldsymbol{\epsilon} = (0{,}0)$, ξ=random(0,1), ζ=random(0,1))
- If shepherd visible ($\|\boldsymbol{S} - \boldsymbol{A}_i\| \leq r_s$)
  - try to move away from the shepherd ($\mathbf{R}_i^s = \mathbf{A}_i - \mathbf{S}$) and toward the centre of mass of *n* nearest neighbours ($\mathbf{C}_i = \mathbf{LCM}_i - \mathbf{A}_i, \mathbf{LCM}_i = \frac{1}{n}\sum_{j=1}^{n} \mathbf{A}_j$)

$$\mathbf{S} = \mathbf{S} + \delta_s\widehat{\mathbf{H}}$$

Shepherd behaviour

- If centre of mass of sheep is within a certain distance of the origin the task is completed
- If too close to a sheep stop ($\exists j: \|\mathbf{A}_i - \mathbf{S}\| < 3r_a \Rightarrow \delta_s = 0$)
- If sheep are collected ($\|\mathbf{GCM} - \mathbf{A}_i\| < r_a N^{2/3}, \mathbf{GCM} = \frac{1}{\mathrm{N}}\sum_{i=1}^{N} \mathbf{A}_j, \forall i = 1, \dots, N$) drive the them towards the goal ($\mathbf{H} = \mathbf{P}_d - \mathbf{S}, \mathbf{P}_d = \mathbf{GCM} + r_a\sqrt{N}\widehat{\mathbf{V}}_1, \mathbf{V}_1 = \mathbf{GCM} - \mathbf{G}$)
- If sheep not collected ($\exists j: \|\mathbf{GCM} - \mathbf{A}_j\| \geq r_a N^{2/3}$) collect the wandering sheep ($\mathbf{H} = \mathbf{P}_c - \mathbf{S}, \mathbf{P}_c = \mathbf{A}_j + r_a\widehat{\mathbf{V}}_2, \mathbf{V}_2 = \mathbf{A}_j - \mathbf{GCM}$)

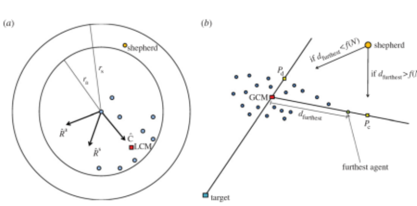


Figure 1. Interaction rules for the agents and the shepherd. (*a*) The agents are attracted to the LCM of their *n* nearest neighbours ($\hat{C}$), repelled from other agents within a distance of r_a ($\hat{\vec{R}}^a$) and repelled from the shepherd if it is within a distance of r_s ($\hat{\vec{R}}^s$). The new heading of the focal agent H' is a linear combination of the three vectors $\hat{C}$, $\hat{\vec{R}}^a$ and $\hat{\vec{R}}^s$, weighted by the corresponding model parameters c, ρ_a, ρ_s, plus a small inertia term $d\hat{H}$ and a small noise term $e\epsilon$. (*b*) In each time step, the shepherd does one out of three things depending on the position of the agents. If the shepherd is within $3r_a$ from any agent, its speed is set to zero. Otherwise, if all agents are within a distance $f(N)$ from the GCM of the agents, then the shepherd aims towards the driving position P_d directly behind the flock relative to the target. Finally, if at least one sheep is further away than $f(N)$ from the GCM, then the shepherd aims for the collecting position P_c directly behind the furthest away sheep relative to the GCM. (Online version in colour.)

Table 1. The parameters of the model. Notation, description and typical values used in simulations.

parameter	description	typical values
L	side length of initial square field	150 m
agent parameters		
N	total number of agents	$1 - 201$
n	number of nearest neighbours	$1 - 200$
r_s	shepherd detection distance	65 m
r_a	agent to agent interaction distance	2 m
ρ_a	relative strength of repulsion from other agents	2
c	relative strength of attraction to the n nearest neighbours	1.05
ρ_s	relative strength of repulsion from the shepherd	1
h	relative strength of proceeding in the previous direction	0.5
e	relative strength of angular noise	0.3
δ	agent displacement per time step	1 m ts^{-1}
p	probability of moving per time step while grazing	0.05
shepherd parameters		
δ_s	shepherd displacement per time step	1.5 m ts^{-1}
P_d	driving position	$r_a\sqrt{N}$ m behind the flock
P_c	collecting position	r_a m behind the furthest agent
e	relative strength of angular noise	0.3
for local shepherd		
n_s	number of nearest agents the local shepherd operates on	20
β	blind angle behind the shepherd	$\pi/2$

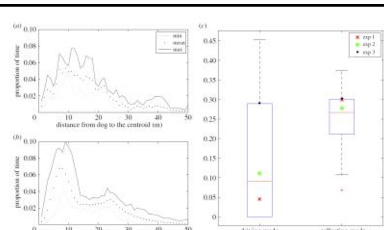


Figure 5. Comparison of the model with data for 46 sheep. (*a*) The proportion of time the dog spent at a certain distance from the GCM of the sheep (length of vector 5 in figure 4) over the three trials. (*b*) The proportion of time the shepherd spent at a certain distance from the GCM of the agents over 100 simulations. The overall shape of the distance distributions in experiments and simulations agree and in particular both exhibit a peak at around 10 m. (*c*) The proportion of time the dog/shepherd spent in driving mode (directly behind the flock relative to the target) and in collecting mode (on the same or opposite side of the flock as the furthest sheep). The boxplots illustrate the proportion of time the shepherd spent in driving or collecting mode over 100 simulations and the points represent the proportion of time the dog spent driving or collecting during three experimental trials. The parameters used in the simulation are the typical values listed in table 1 except for $n = 46$, $r_s = 75$ m, $\rho_a = 1$ and $e = 0.1$. The experimental data can be seen in the electronic supplementary material, video 2, and simulations with these parameter values in video 3. (Online version in colour.)

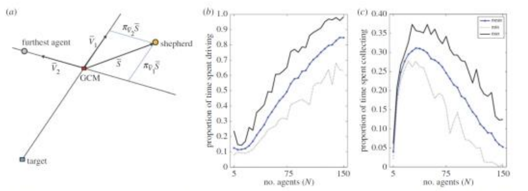


Figure 4. Projections used to define the driving and collecting modes and how the proportion of time spent driving and collecting depends on the number of agents. (*a*) How the centroid vector v_1, the furthest agent vector v_2 and shepherd vector S are set up. Three measurements related to these vectors will be used in comparing the model to the sheep data (figure 5). The projection of S on v_1 denoted by $\pi_{v_1}S$, the projection of S on v_2 denoted by $\pi_{v_2}S$ and the length of S. (*b*) Proportion of time the shepherd spends driving ($\pi_{v_1}S > 0.95$) as a function of group size (N) in the global case ($n = N - 1$) over 100 simulations. (*c*) Proportion of time spent collecting ($|\pi_{v_2}S| > 0.95$) as a function of group size (N) in the global case over 100 simulations. The other parameters are the typical values listed in table 1. (Online version in colour.)

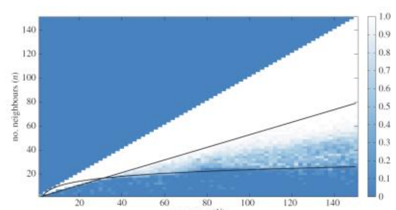
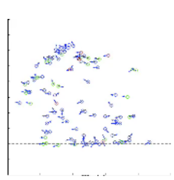


Figure 3. Proportion of successful shepherding events within 8000 time steps as a function of $N \in [2, 150]$ and nearest neighbours $n \in [1, N - 1]$. We see that in the global case $n = N - 1$ and down to roughly $n = 0.53N$ the algorithm is always successful. For $N > 30$, there is a transition region below the line $n = 0.53N$ and above $n \approx 3\log(N)$ where the probability of success drops from 1 to rare at a rate that is decreasing with N. Finally, in the region under the curve $\min(0.53N, 3\log(N))$ success is rare and sporadic. The other parameters are the typical values listed in table 1 except for $r_s = 45$. (Online version in colour.)

Ginelli et al. 2014 doi: 10.1098/rsif.2014.0719

Based on quantitative field observations of large groups of Merino sheep. While grazing, these sheep must balance two competing needs: (i) the maximization of individual foraging space and (ii) the protection from predators offered by a large dense group.

Universe: $L \times L$ field, N sheep ($\mathbf{r}_i, i = 1, \dots, N$)



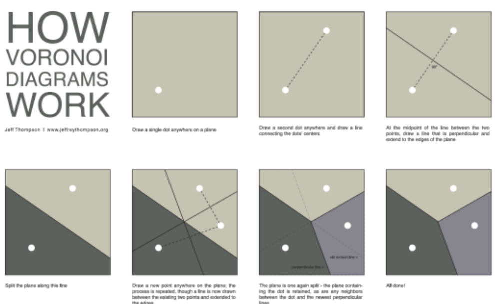
$$\mathbf{r}_i^{t+\Delta t} = \mathbf{r}_i^t + \Delta t\, v(q_i^t)\, \mathbf{s}_i^{t+\Delta t}, \qquad [1]$$

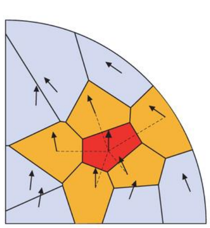
$$\theta_i^{t+\Delta t} = \mathrm{Arg}\left[\sum_{j \in \mathcal{M}_i} \mathbf{s}_j^t\right] + \psi_i^t \quad (\text{if } q_i^t = 1), \qquad [2]$$

$$\theta_i^{t+\Delta t} = \mathrm{Arg}\sum_{j \in \mathcal{V}_i}\left[\delta_{2,q_j^t}\, \mathbf{s}_j^t + \beta f\left(r_{ij}^t\right)\mathbf{e}_{ij}^t\right] \quad (\text{if } q_i^t = 2) \qquad [3]$$

Sheep behaviour

- Heading $\mathbf{s}_i^t = [\cos\theta_i^t, \sin\theta_i^t]$, distance $r_{ij}^t = \|\mathbf{r}_j - \mathbf{r}_i\|$, unit vector $\mathbf{e}_{ij}^t = (\mathbf{r}_j - \mathbf{r}_i)/\|\mathbf{r}_j - \mathbf{r}_i\|$
- Three states: idle ($q_i^t = 0, v(0) = 0$), walking ($q_i^t = 1, v(1) = 0.15$), running ($q_i^t = 2, v(2) = 1.5$)
- If walking align heading with *metric* neighbours ($\mathcal{M}_i = \{j: r_{ij} < r_o\}$), but not always ($\psi_i^t$ random uniform "noise" $[-\eta\pi, \eta\pi]$)
- If running align heading interact (stay as close as possible and at a safe distance) with all *topologic* neighbours ($\mathcal{V}_i$ = first shell of Voronoi neighbours, $\beta f(r_{ij}^t)\hat{\mathbf{e}}_{ij}^t, f(r_{ij}^t) = \min(1, (r - r_e)/r_e)$) and align heading with running topologic neighbours ($\delta_{2,q_j^t}\mathbf{s}_j^t$)





Gautrais J, et al. (2012) doi: 10.1371/journal.pcbi.1002678

State transitions

$$p_{0\to 1}(i,t) = \frac{1 + \alpha n_1^r(i)}{\tau_{0\to 1}}, \quad p_{1\to 0}(i,t) = \frac{1 + \alpha n_0^r(i)}{\tau_{1\to 0}}, \qquad [4]$$

where $\tau_{0\to 1}$ and $\tau_{0\to 1}$ are spontaneous transition times, α measures the strength of mimetic effects, and $n_0^r(i)$, $n_1^r(i)$ is the number of stationary and walking metric neighbors, respectively.

The transitions to and from the running state are similar, but they depend on the number m_R of running topological neighbors, with the allelomimetic effect strengthened by an exponent $\delta > 1$,

$$p_{0,1\to 2}(i,t) = \frac{1}{\tau_{0,1\to 2}}\left[\frac{\ell_i^t}{d_R}(1 + \alpha m_R^t(i))\right]^{\delta}, \qquad [5]$$

where ℓ_i^t is the mean distance to all topological neighbors of sheep i, and d_R is some characteristic length scale. The ratio between these two scales ensures that spread-out groups are much more likely to trigger a packing event than high-density ones.

Finally, for simplicity, running sheep can only transit to the stationary state with a rate $p_{2\to 0}(i,t)$ enhanced by m_S, the number of their stopping topological neighbors, i.e., those that switched from running to stationary in the previous time step,

$$p_{2\to 0}(i,t) = \frac{1}{\tau_{2\to 0}}\left[\frac{d_S}{\ell_i^t}(1 + \alpha m_S^t(i))\right]^{\delta}, \qquad [6]$$

where $d_S < d_R$ is a second characteristic length. The positive feed-

$$P = 1 - e^{-p\Delta t}$$