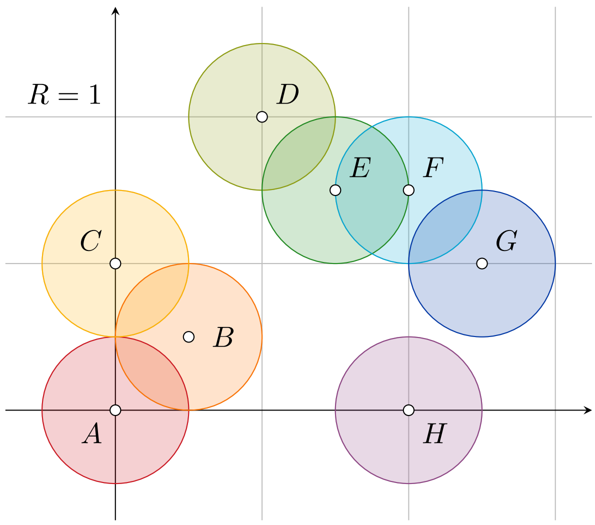
Computational topology
Lab work, 5th week

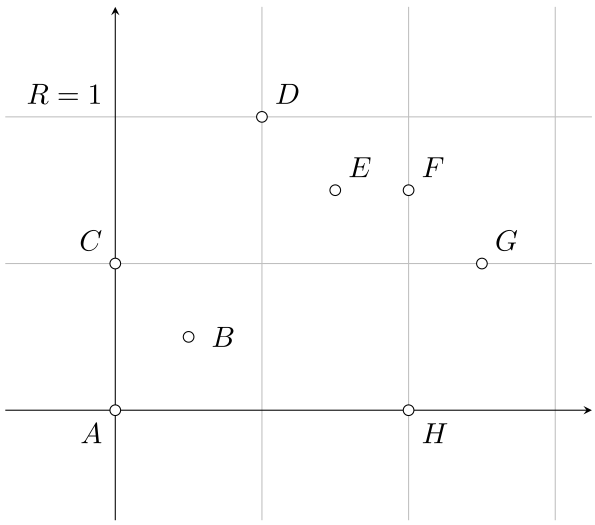
1. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Vietoris-Rips complex $\mathrm{Rips}(S,R)$ for

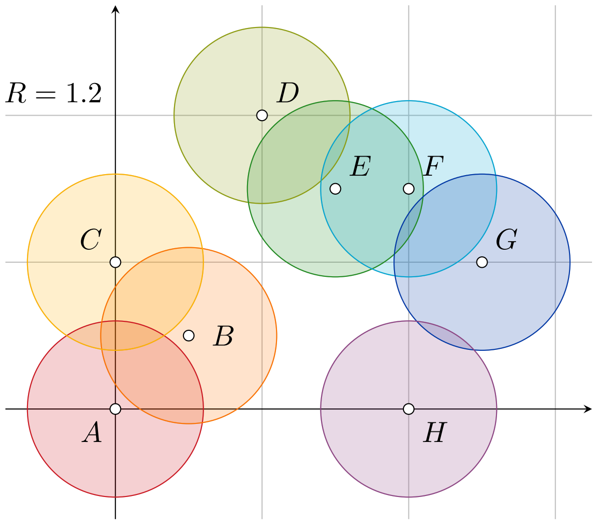
$$\text{(a) } R = 1, \quad \text{(b) } R = 1.2, \quad \text{(c) } R = 1.75.$$

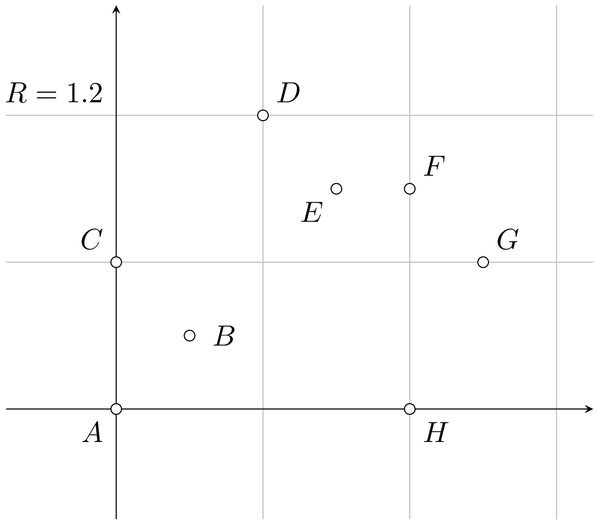
In each case list all the simplices and determine its dimension.

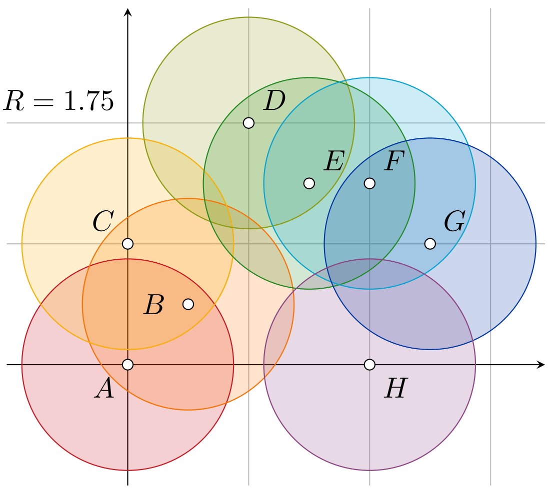
If there is a sensor placed at each point of S and all sensors can detect points that are at distance 1.75 or less, is the area covered by the sensors connected? Does it contain any holes?

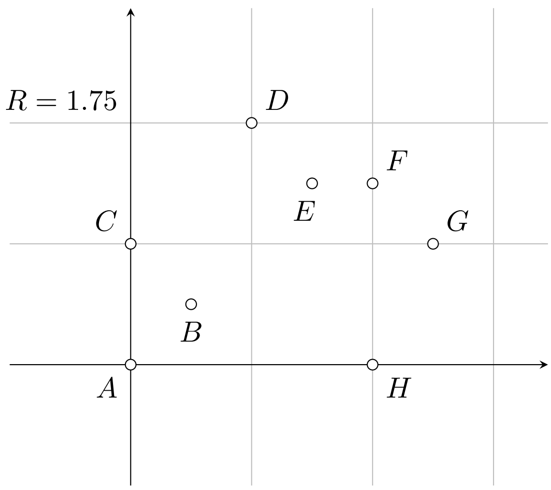








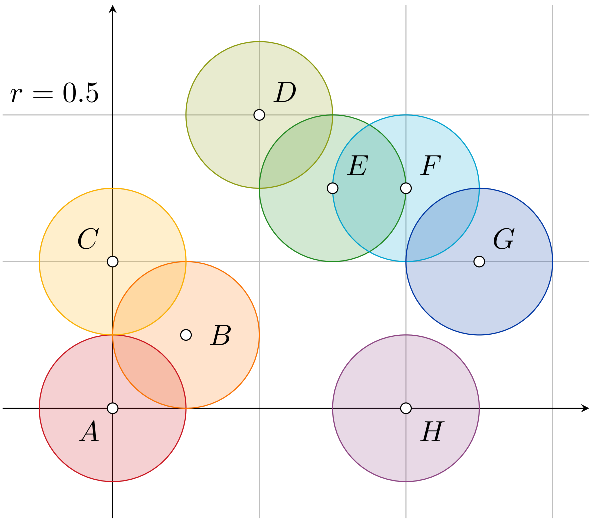


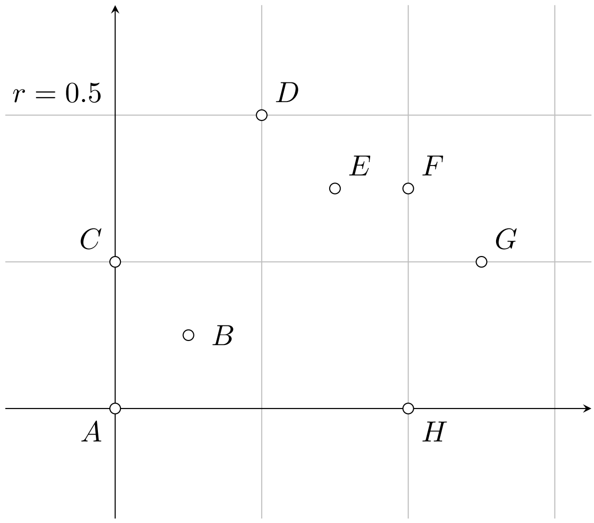


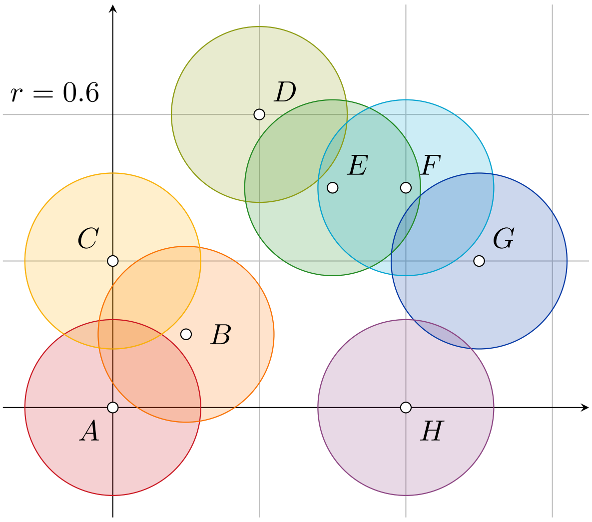
2. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Čech complex $\mathrm{Cech}(S,r)$ for

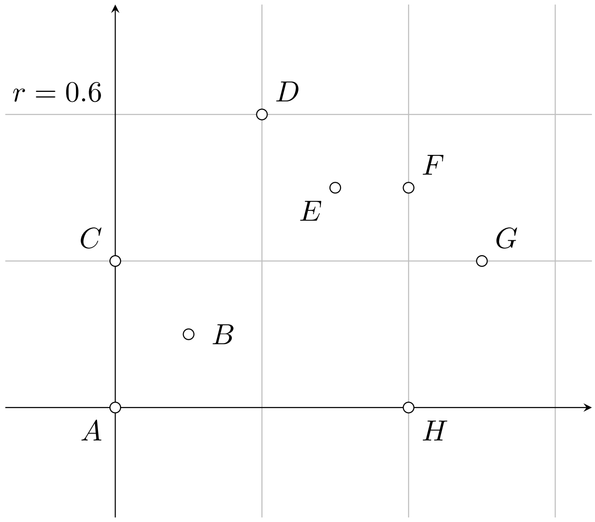
$$\text{(a) } R = 0.5, \quad \text{(b) } R = 0.6, \quad \text{(c) } R = 0.875.$$

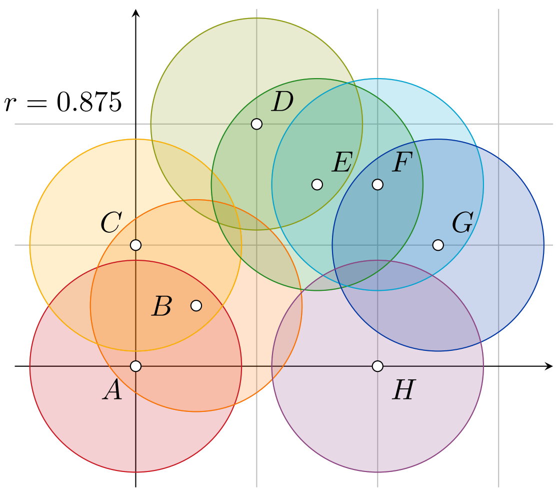
In each case list all the simplices and determine its dimension.

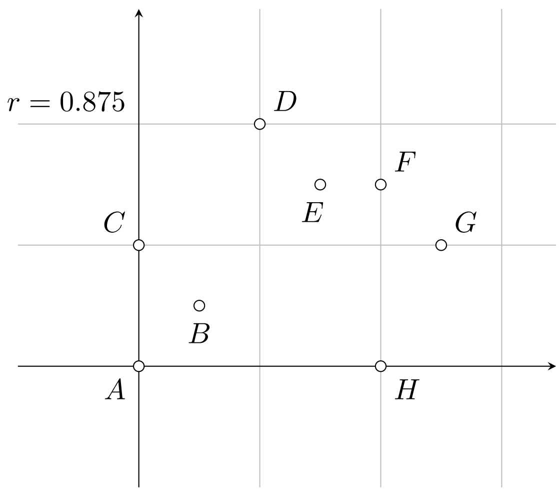




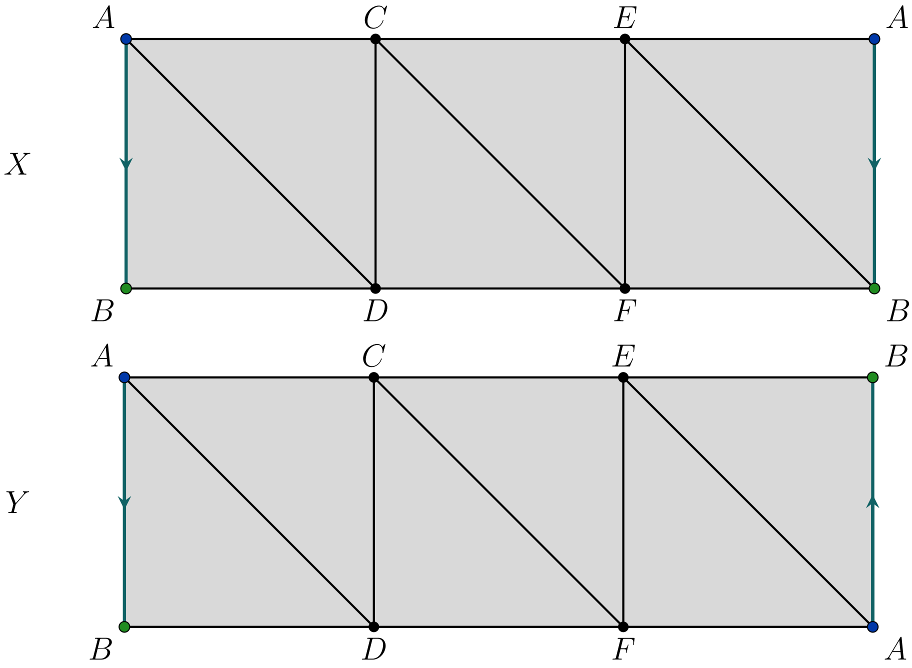








3. Given the following triangulations of the cylinder X and the Moebius band Y, find a sequence of elementary collapses that simplifies them as much as possible, then compute the homology groups $H_*(X)$ and $H_*(Y)$.



4. For the simplicial complex X in the figure below

   (a) write down the chain groups $\mathcal{C}_n$,

   (b) determine the boundary homomorphisms $\partial_n \colon \mathcal{C}_n \to \mathcal{C}_{n-1}$,

   (c) find the cycles $Z_n = \ker\partial_n$,

   (d) find the boundaries $B_n = \mathrm{im}\partial_n$,

   (e) determine the simplicial homology groups with $\mathbb{Z}$ coefficients, $H_n(X;\mathbb{Z})$,

   (f) determine the simplicial homology groups with $\mathbb{Z}_2$ coefficients, $H_n(X;\mathbb{Z}_2)$,

   (g) determine the Betti numbers of X and

   (h) compute the Euler characteristic of X.