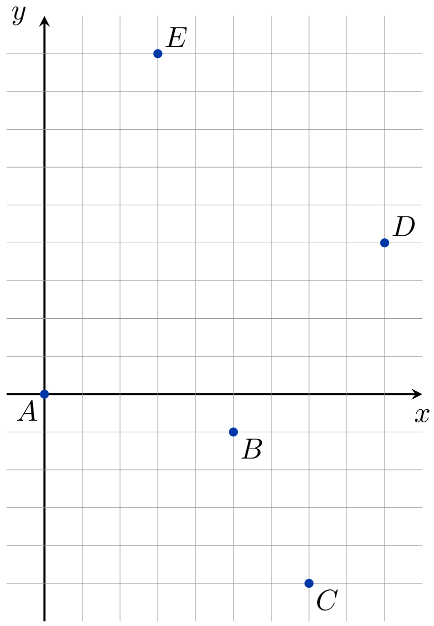
# Topological Data Analysis
## Lab work, $6^{\text{th}}$ week

1. Let $S = \{A(0,0), B(5,-1), C(7,-5), D(9,4), E(3,9)\} \subset \mathbb{R}^2$.

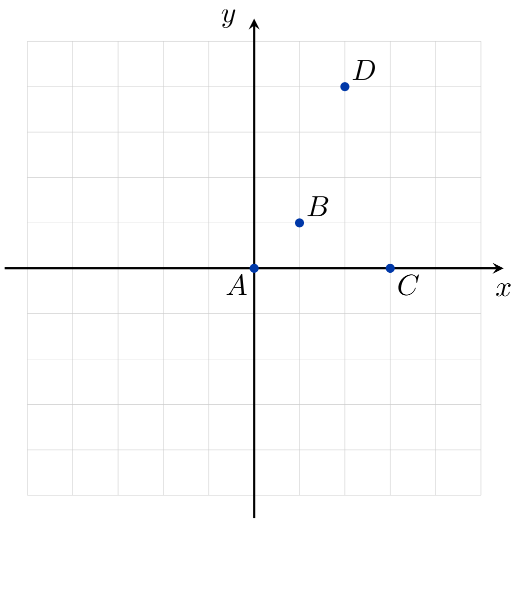
   (a) Construct the triangulations $\mathcal{T}_1$ and $\mathcal{T}_2$ of $S$ using vertical line sweep from left to right and the horizontal line sweep upwards.

   (b) We can get the Delaunay triangulation on $S$ by flipping certain edges. How many edge flips are necessary to produce a Delaunay triangulation from $\mathcal{T}_1$? From $\mathcal{T}_2$?

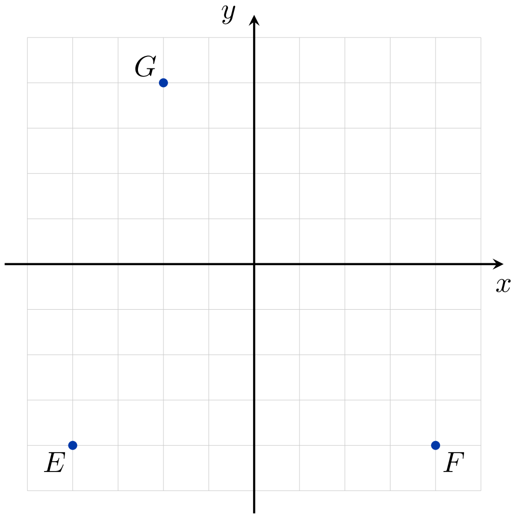
   (c) Draw the corresponding Voronoi diagram. Is it unique?




2. *Hermes messenger service, Ltd.* has distribution centres placed at $A(0,0)$, $B(1,1)$, $C(3,0)$ and $D(2,4)$. Divide the $[-5,5] \times [-5,5]$ square into service areas that ensure the fastest packet delivery.



Their competition, *Mercury post*, has the distribution centres located at $E(-4,-4)$, $F(4,-4)$ and $G(-2,4)$, but the center at $E$ can only deliver within a 7 unit radius and the center at $G$ only within a 6 unit radius. The center at $F$ has more employees and uses bike messengers so they can deliver within an 10 unit radius. How should they split the service area?




3. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Vietoris-Rips complex $\mathrm{VR}(S,R)$ for

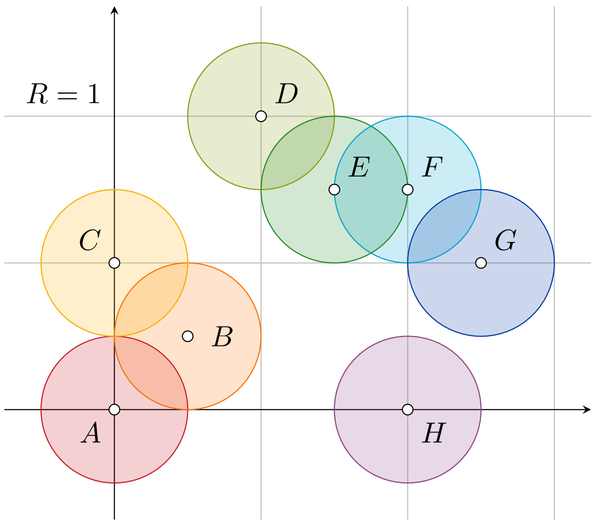
   (a) $R = 1$,

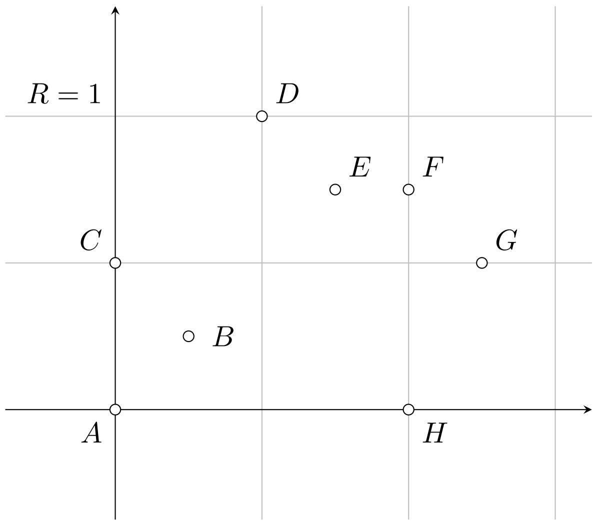
   (b) $R = 1.2$,

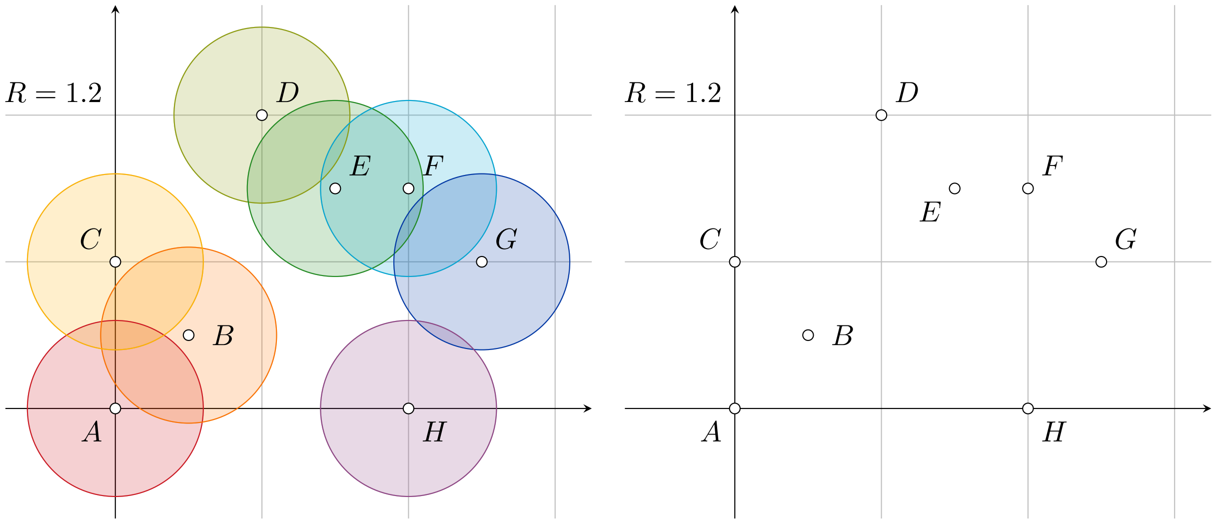
   (c) $R = 1.75$.

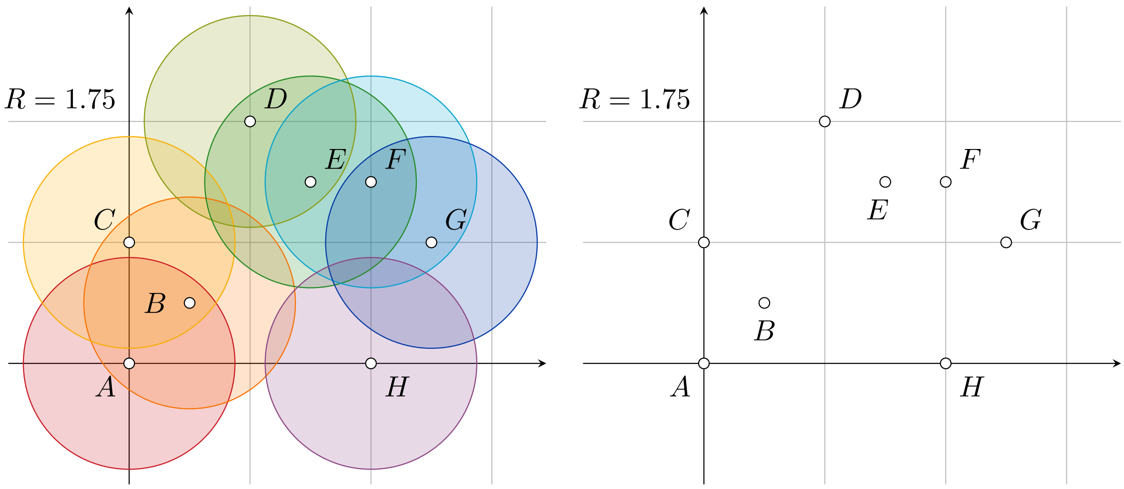
   In each case list all the simplices and determine its dimension.

   Assuming there is a sensor placed at each point of $S$ and all sensors can detect points that are at distance 1.75 or less, is the area covered by the sensors connected? Does it contain any holes?





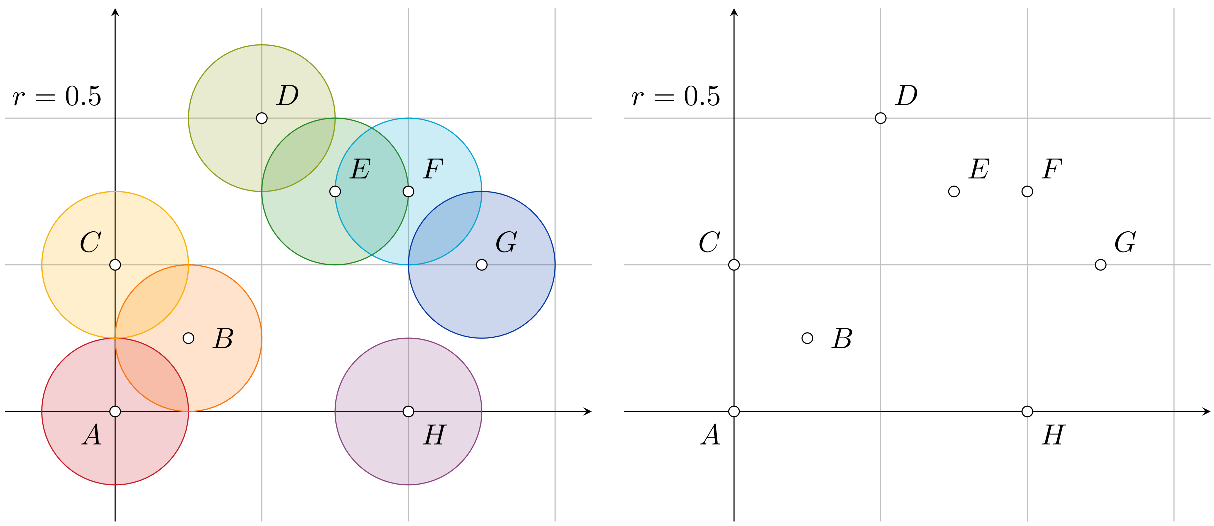

4. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Čech complex $\mathrm{C}(S,r)$ for

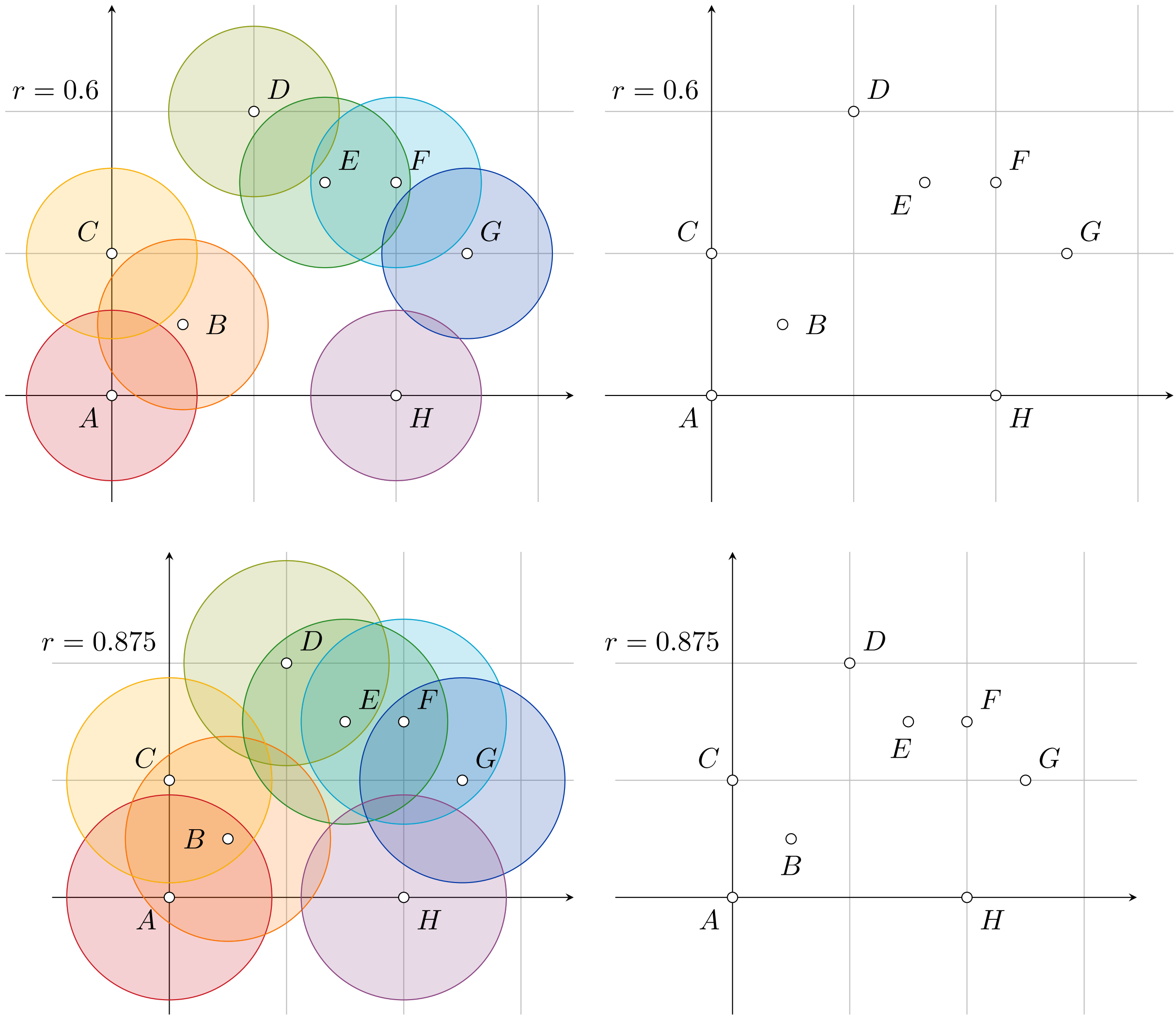
   (a) $r = 0.5$,

   (b) $r = 0.6$,

   (c) $r = 0.875$.

   In each case list all the simplices and determine its dimension.




r = 0.6
A
B
C
D
E
F
G
H
r = 0.6
A
B
C
D
E
F
G
H
r = 0.875
A
B
C
D
E
F
G
H
r = 0.875
A
B
C
D
E
F
G
H