Computational topology

Lab work 12

1. Two different monotonic functions are given on the simplicial complex X:

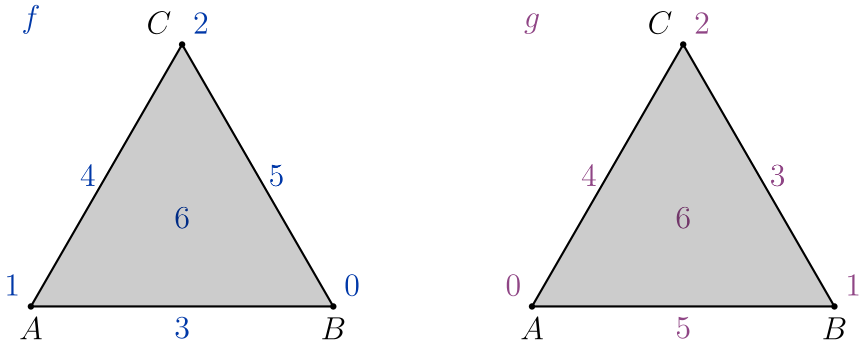
$$\begin{aligned} f &= \{(A,1),\ (B,0),\ (C,2),\ (AB,3),\ (AC,4),\ (BC,5),\ (ABC,6)\}, \\ g &= \{(A,0),\ (B,1),\ (C,2),\ (AB,5),\ (AC,4),\ (BC,3),\ (ABC,6)\}. \end{aligned}$$

   (a) Create the corresponding filtrations of subcomplexes.

   (b) Draw the barcode diagrams and the persistence diagrams in dimensions 0 and 1.

   (c) Construct the boundary matrices D_f and D_g from the two filtrations.

   (d) Use the matrix reduction to calculate persistence.

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R = D
for j = 1 to m:
    while there exists j_0 < j with low(j_0) = low(j):
        add column R[:, j_0] to column R[:, j]
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2. Let $A = (1, 3)$, $B = (2, 4)$, $C = (1, 2)$ $D = (2, 5)$ and $E = (1, \infty)$. For each of the pairs X_i, Y_i of persistent diagrams given below

   - find all bijections $\eta\colon X_i \to Y_i$,
   - determine $\|x - \eta(x)\|_\infty$ for each bijection and for all $x \in X_i$ and
   - calculate the bottleneck distances $W_\infty(X_i, Y_i)$ and Wasserstein distances $W_q(X_i, Y_i)$ for $q = 1, 2$.

   (a) $X_1 = \Delta \cup \{A\}$, $Y_1 = \Delta \cup \{B\}$,

   (b) $X_2 = \Delta \cup \{A, B\}$, $Y_2 = \Delta \cup \{C\}$,

   (c) $X_3 = \Delta \cup \{A, B\}$, $Y_3 = \Delta \cup \{C, D\}$,

   (d) $X_4 = \Delta \cup \{A, E\}$, $Y_4 = \Delta \cup \{C\}$.

The **bottleneck distance** between persistence diagrams X and Y:

$$W_\infty(X, Y) = \inf_{\eta\colon X \to Y} \left(\sup_{x \in X} \|x - \eta(x)\|_\infty \right).$$

The **Wasserstein distance** for all $q \in \mathbb{R}$:

$$W_q(X,Y) = \left(\inf_{\eta\colon X \to Y} \sum_{x \in X} ||x - \eta(x)||_\infty^q \right)^{\frac{1}{q}}.$$