1 Amortized analysis

1.1 Comparing methods

Perform a sequence of n operations on a data structure in which almost every operation has a constant cost $c_i = 1$. Exceptions are operations c_i where $i = 2^k$ and $k \in \mathbb{N}$, these operations costs $c_i = i$. Show that amortized cost per operation is constant using:

a) Aggregate method

b) Accounting method

c) Potential method

1.2 Full analysis of dynamic tables

You are given a dynamic table as shown on lectures which doubles its size when it's full but can also contratct to save space. Such dynamic tables use both dynamic expansions and contractions. Let's define num_i as the number of elements in the table after i-th operation and size_i the size of dynamic table after i-th operation. Load factor is defined as $\alpha_i = \frac{\text{num}_i}{\text{size}_i}$.

a) Suppose you contract the table as soon as the load factor falls to $\alpha_i < \frac{1}{2}$. Show that in this case the worst time per operation is $O(n)$.

b) Find a loading factor α at which you need to halve the table so that cost per operation remains constant. Prove it using potential method.