3.4. Linearne preslikave

Motivacija: Naj bo $A \in \mathbb{R}^{m\times n}$. Oglejmo si preslikavo

$$\varphi: \mathbb{R}^n \to \mathbb{R}^m$$
$$\vec{x} \mapsto A\vec{x}$$

"$\vec{x}$ se slika v $A\vec{x}$"

V posebnem: $m=n=2$, $A \in \mathbb{R}^{2\times 2}$, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\varphi: \mathbb{R}^2 \to \mathbb{R}^2$$
$$\vec{x} \mapsto A\vec{x} = \vec{y}$$

$$\vec{y} = \varphi(\vec{x}) = A\vec{x} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{bmatrix} = x_1\begin{bmatrix} a \\ c \end{bmatrix} + x_2\begin{bmatrix} b \\ d \end{bmatrix}$$

$\varphi(\vec{\imath}) = \varphi\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} a \\ c \end{bmatrix}$ 1. stolpec matrike A

$\varphi(\vec{\jmath}) = \varphi\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} b \\ d \end{bmatrix}$ 2. stolpec matrike A

Tudi v splošnem, ko $A = \left[A^{(1)}\ A^{(2)} \dots A^{(n)}\right] \in \mathbb{R}^{m\times n}$:

stolpci matrike A

$$\varphi(\vec{e}_1) = \left[A^{(1)}\ A^{(2)} \dots A^{(n)}\right]\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = A^{(1)} \leftarrow \text{1. stolpec } A$$

$\varphi(\vec{e}_2) = A^{(2)} \quad \leftarrow$ 2. stolpec A

$\vdots$

$\varphi(\vec{e}_n) = A^{(n)} \quad \leftarrow$ n-ti stolpec A

Definicija: Naj bosta U in V vektorska prostora. Preslikavi $\varphi: U \to V$ pravimo **linearna**, če

1) $\varphi(u_1 + u_2) = \varphi(u_1) + \varphi(u_2)$ za $\forall u_1, u_2 \in U$
2) $\varphi(\alpha u) = \alpha\varphi(u)$ za $\forall u \in U$ in $\forall \alpha \in \mathbb{R}$.

Ali ekvivalentno: $\varphi(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1\varphi(u_1) + \alpha_2\varphi(u_2)$ $\forall u_1, u_2 \in U$, $\forall \alpha_1, \alpha_2 \in \mathbb{R}$

Primeri: 1) $A \in \mathbb{R}^{m\times n}$. Pokažimo, da je $\varphi: \mathbb{R}^n \to \mathbb{R}^m$, $\vec{x} \mapsto A\vec{x}$ linearna preslikava.

Način A: 1) $\varphi(\vec{x}+\vec{y}) \overset{def}{=} A(\vec{x}+\vec{y}) \overset{distr.}{=} A\vec{x} + A\vec{y} \overset{def}{=} \varphi(\vec{x}) + \varphi(\vec{y})$ za $\forall \vec{x}, \vec{y} \in \mathbb{R}^n$

2) $\varphi(\alpha\vec{x}) \overset{def}{=} A(\alpha\vec{x}) = \alpha\cdot(A\vec{x}) = \alpha\,\varphi(\vec{x})$ za $\forall \vec{x} \in \mathbb{R}^n$, $\forall \alpha \in \mathbb{R}$

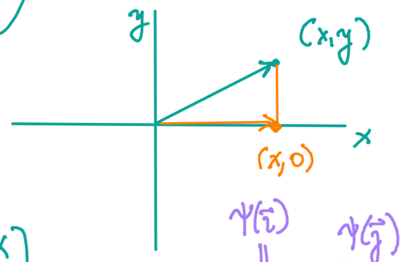
1)+2) φ je linearna preslikava

Način B: $\varphi(\alpha\vec{x}+\beta\vec{y}) \overset{def}{=} A(\alpha\vec{x}+\beta\vec{y}) \overset{distr.}{=} A(\alpha\vec{x}) + A(\beta\vec{y}) =$
$= \alpha A\vec{x} + \beta A\vec{y}$
$= \alpha\,\varphi(\vec{x}) + \beta\,\varphi(\vec{y})$

$\Rightarrow \varphi$ je linearna preslikava.

Primer ② $\psi : \mathbb{R}^2 \to \mathbb{R}^2$ je pravokotna projekcija na x-os.

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ 0 \end{bmatrix}$$



$$\psi\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\psi\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A \begin{bmatrix} x \\ y \end{bmatrix}$$

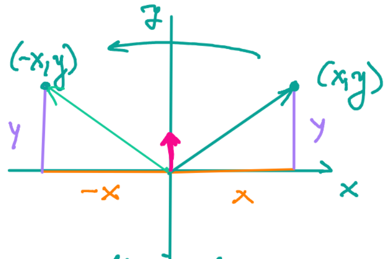
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \psi(\vec{i}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \psi(\vec{j}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \psi$ je linearna preslikava

Nauk: če znamo preslikavo zapisati kot matrično množenje, je ta preslikava linearna.

③ $\vartheta : \mathbb{R}^2 \to \mathbb{R}^2$ je zrcaljenje čez y-os

$$\vartheta\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix}$$



$$\vartheta\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

množenje z matriko $\Rightarrow \vartheta$ je linearna

$$A_\vartheta = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \vartheta(\vec{i}) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad \vartheta(\vec{j}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

④ Primer nelinearne preslikave.

$$Q : \mathbb{R}^2 \to \mathbb{R}^2$$
$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ y^2 \end{bmatrix}$$

Zakaj ni linearna?

$$Q\left(\alpha\begin{bmatrix} x \\ y \end{bmatrix}\right) = Q\left(\begin{bmatrix} \alpha x \\ \alpha y \end{bmatrix}\right) = \begin{bmatrix} \alpha x \\ (\alpha y)^2 \end{bmatrix} =$$
$$= \alpha \begin{bmatrix} x \\ \alpha y^2 \end{bmatrix} \neq \alpha\, Q\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

$$Q\left(2\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \overset{\downarrow}{=} 2\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$
$$2\,Q\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$\Rightarrow$ 2) ne drži
$\Rightarrow Q$ ni linearna

⑤ $\mathcal{D}: \left\{\begin{matrix}\text{zvezno}\\ \text{odv. f.}\end{matrix}\right\} \longrightarrow \{\text{zvezna } f.\}$

$\mathcal{D}(f) = f' \qquad f \longmapsto f'$

1) $\mathcal{D}(f+g) \overset{def}{=} (f+g)' \overset{OMA}{=} f'+g' \overset{def}{=} \mathcal{D}(f) + \mathcal{D}(g)$

2) $\mathcal{D}(\alpha f) \overset{def}{=} (\alpha f)' \overset{OMA}{=} \alpha f' \overset{def}{=} \alpha \mathcal{D}(f)$

$\Big\}$ $\mathcal{D}$ je linearna preslikava

⑥ $vec: \mathbb{R}^{m\times n} \longrightarrow \mathbb{R}^{mn}$ (vectorization / vektorizacija matrik)

$$A = [\underbrace{A^{(1)}}_{\in\mathbb{R}^m} \ldots \underbrace{A^{(n)}}_{\in\mathbb{R}^m}] \longmapsto \left.\begin{bmatrix} A^{(1)} \\ A^{(2)} \\ \vdots \\ A^{(n)} \end{bmatrix}\begin{matrix}\}\mathbb{R}^m \\ \}\mathbb{R}^m \\ \vdots \\ \}\mathbb{R}^m\end{matrix}\right\} \mathbb{R}^{nm}$$

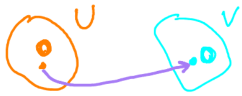
$$vec \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

Ali je vec linearna preslikava?

$vec(\alpha A + \beta B) = \alpha\, vec(A) + \beta\, vec(B)$ ← preverite doma

Lastnosti linearnih preslikav:

U, V vekt. prostora $\quad T: U \to V$ linearna preslikava

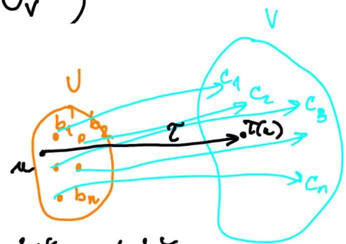


① $T(\underset{\substack{\uparrow\\ U}}{0_U}) = \underset{\substack{\uparrow\\ V}}{0_V}$

(Zakaj? $T(0_U) = T(0 \cdot \underset{\substack{\uparrow\\ U}}{u}) \overset{2)}{=} 0 \cdot \underbrace{T(u)}_{\substack{\uparrow\\ V}} = 0_V$)

② Denimo, da je $\mathcal{B} = \{b_1, \ldots, b_n\}$ baza vekt. pr. U.

Naj $c_1 = T(b_1),\ c_2 = T(b_2), \ldots, c_n = T(b_n)$



Naj $u \in U$. Potem je $u = \alpha_1 b_1 + \ldots + \alpha_n b_n$, α_i enolično določeni.

$\underline{T(u)} = T(\alpha_1 b_1 + \ldots + \alpha_n b_n) \Leftarrow$ lastnost lin. preslikave

$= \alpha_1 \underbrace{T(b_1)}_{= c_1} + \ldots + \alpha_n \underbrace{T(b_n)}_{= c_n}$ → lin. kombinacija $T(b_1), \ldots, T(b_n)$

③ Kako zapišemo matriko, ki pripada linearni preslikavi?

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$\mathcal{S}_n = \{\vec{e}_1, \ldots, \vec{e}_n\}$ stand. baza $\mathbb{R}^n$

$\mathcal{S}_m = \{\vec{f}_1, \ldots, \vec{f}_m\}$ stand. baza $\mathbb{R}^m$

$$\vec{e}_i = \begin{array}{r} \\ i \to \\ \\ \end{array}\left.\begin{bmatrix} \\ 1 \\ \\ \end{bmatrix}\right\}n \in \mathbb{R}^n \qquad \vec{f}_j = \begin{array}{r} \\ j \to \\ \\ \end{array}\left.\begin{bmatrix} \\ 1 \\ \\ \end{bmatrix}\right\}m \in \mathbb{R}^m$$

1. Poračunamo $T(\vec{e}_1), \ldots, T(\vec{e}_n)$
2. $T(\vec{e}_1) = \alpha_{11} \vec{f}_1 + \alpha_{21} \vec{f}_2 + \ldots + \alpha_{m1} \vec{f}_m$

$T(\vec{e}_2) = \alpha_{12} \vec{f}_1 + \alpha_{22} \vec{f}_2 + \ldots + \alpha_{m2} \vec{f}_m$

$\vdots$

$T(\vec{e}_n) = \alpha_{1n} \vec{f}_1 + \alpha_{2n} \vec{f}_2 + \ldots + \alpha_{mn} \vec{f}_m$ ←

3.

$$A_\tau = \underset{\text{lin. presl.}\ \nearrow\ \ \nearrow\ \text{bazi}}{A_{\tau, S_n, S_m}} = \begin{array}{c} \overbrace{\begin{array}{cccc} \vec e_1 & \vec e_2 & \cdots & \vec e_n \end{array}}^{\mathbb{R}^n} \\ \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix} \end{array} \left.\begin{array}{c} \vec f_1 \\ \vec f_2 \\ \vdots \\ \vec f_m \end{array}\right\} \mathbb{R}^m \quad \in \mathbb{R}^{m \times n}$$

to matriko imenujemo matrika, ki pripada lin. presl. τ iz baze S_n v bazo S_m.

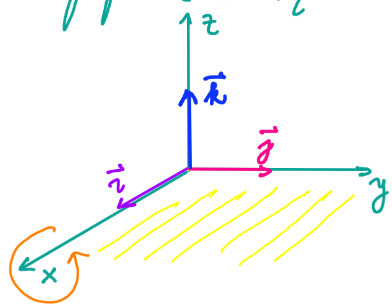
Naj $\vec u \in \mathbb{R}^n$: $\vec u = \beta_1 \vec e_1 + \dots + \beta_n \vec e_n$. Ker $\{\vec e_1, \dots, \vec e_n\}$ stand. baza, je $\vec u = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$

$$\tau(\vec u) = \beta_1 \underbrace{\tau(\vec e_1)}_{A^{(1)}} + \dots + \beta_n \underbrace{\tau(\vec e_n)}_{A^{(n)}} = A^{(1)}\beta_1 + A^{(2)}\beta_2 + \dots + A^{(n)}\beta_n =$$

$$= \underbrace{\begin{bmatrix} A^{(1)} & A^{(2)} & \cdots & A^{(n)} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}}_{\vec u} = A \vec u$$

Primer: Naj bo τ preslikava $\mathbb{R}^3 \to \mathbb{R}^3$, ki vsak vektor/točko najprej zasuka okoli x osi za kot $\frac{\pi}{2}$, nato pa še prezrcali preko ravnine $z=0$.

a) Vemo, da je τ lineama. Poiščimo matriko, ki pripada τ iz stand. baze $\{\vec i, \vec j, \vec k\}$ v stand. bazo $\{\vec i, \vec j, \vec k\}$.



$$\begin{array}{lllll} \vec i & \xrightarrow[\text{okoli x-osi za } \frac{\pi}{2}]{\text{zasuk}} & \vec i & \xrightarrow[\text{čez } z=0]{\text{zrcaljenje}} & \vec i \qquad \tau(\vec i) = \vec i \\ \vec j & \longmapsto & \vec k & \longmapsto & -\vec k \qquad \tau(\vec j) = -\vec k \\ \vec k & \longmapsto & -\vec j & \longmapsto & -\vec j \qquad \tau(\vec k) = -\vec j \end{array}$$

$$A_\tau = A_{\tau, S_3, S_3} = \begin{array}{cc} \begin{array}{ccc} \vec i & \vec j & \vec k \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} & \begin{array}{c} \vec i \\ \vec j \\ \vec k \end{array} \end{array}$$ ← matrika, ki pripada τ

b) Kam se s τ preslika $\vec a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

$$\tau(\vec a) = A_\tau \vec a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$$

Kaj pa v primeru, ko $T: \mathbb{R}^{m\times n} \to \mathbb{R}^{p\times q}$?
$T(E_{ij})$ v stolpce mtr. $\in \mathbb{R}^{(pq)\times(mn)}$.

Primer: Oglejmo si $T: \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$
$A \mapsto A^T$ $\quad T(A)=A^T$

a) Ali je T linearna?
$A,B\in \mathbb{R}^{2\times 2}$, $\alpha,\beta\in\mathbb{R}$: $T(\alpha A+\beta B) \overset{def}{=} (\alpha A+\beta B)^T = \alpha A^T+\beta B^T =$
$= \alpha T(A) + \beta T(B)$

$\Rightarrow T$ je linearna

b) Zapišimo matriko, ki pripada T iz st. baze $\{E_{11}, E_{12}, E_{21}, E_{22}\}$ v st. bazo $\{E_{11}, E_{12}, E_{21}, E_{22}\}$.

$T(E_{11}) = \begin{bmatrix}1&0\\0&0\end{bmatrix}^T = \begin{bmatrix}1&0\\0&0\end{bmatrix} = E_{11} = 1\cdot E_{11} + 0\cdot E_{12} + 0\cdot E_{21} + 0\cdot E_{22}$ $\leftarrow$ 1. stolpec A

koeficienti so

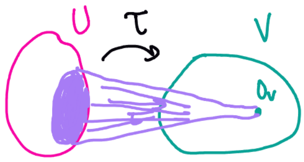
$T(E_{12}) = E_{12}^T = E_{21} = 0\cdot E_{11} + 0\cdot E_{12} + 1\cdot E_{21} + 0\cdot E_{22}$ $\leftarrow$ 2. stolpec A

$T(E_{21}) = E_{21}^T = E_{12}$ $\leftarrow$ 3. stolpec A

$T(E_{22}) = E_{22}^T = E_{22}$ $\leftarrow$ 4. stolpec A

$$A_{T, \mathcal{S}_{2\times 2}, \mathcal{S}_{2\times 2}} = \begin{array}{c c} \begin{matrix} E_{11} & E_{12} & E_{21} & E_{22} \end{matrix} & \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{matrix} E_{11} \\ E_{12} \\ E_{21} \\ E_{22} \end{matrix} \end{array} \in \mathbb{R}^{4\times 4}$$

$T: U \to V$ linearna preslikava, U, V vektorska prostora



Def: Množico

$\ker(T) = \{u\in U;\ T(u) = 0_V\}$ angl. "kernel"

imenujemo jedro preslikave T.
$\hookrightarrow$ vekt. podprostor v U

$T: \mathbb{R}^n \to \mathbb{R}^m$ linearna, $A\in\mathbb{R}^{m\times n}$ matrika, ki pripada T v st. bazah
$T(\vec{u}) = A\vec{u}$

$\ker T = \{\vec{u}\in\mathbb{R}^n;\ T(\vec{u}) = \vec{0}\} =$
$= \{\vec{u}\in\mathbb{R}^n;\ A\cdot\vec{u} = \vec{0}\} = N(A)$

Def: Množico

$\operatorname{im}(T) = \{\vec{v}\in V;\ \vec{v} = T(\vec{u})$ za nek $\vec{u}\in U\}$ angl. "image"

imenujemo slika lin. preslikave T.
$\hookrightarrow$ vekt. podprostor v V

v primeru, ko $T: \mathbb{R}^n \to \mathbb{R}^m$, $T(\vec{u}) = A \cdot \vec{u}$

$$\operatorname{im} T = \{\vec{v} \in \mathbb{R}^m;\ \vec{v} = T(\vec{u}) \text{ za } \vec{u} \in \mathbb{R}^n\} =$$

$$= \{\vec{v} \in \mathbb{R}^m;\ \vec{v} = A\vec{u} \text{ za } \vec{u} \in \mathbb{R}^n\}$$

$A\vec{u} = \vec{v}$ lin sistem

$$= \{\vec{v} \in \mathbb{R}^m;\ \vec{v} \in C(A)\} = C(A)$$

Vemo: za $A \in \mathbb{R}^{m \times n}$ velja $\dim N(A) + \dim C(A) = n$

prevod v lin. presl.

za $T: U \to V$ velja $\boxed{\dim(\ker(T)) + \dim(\operatorname{im}(T)) = \dim U}$

linearna $T: \mathbb{R}^3 \to \mathbb{R}^3$

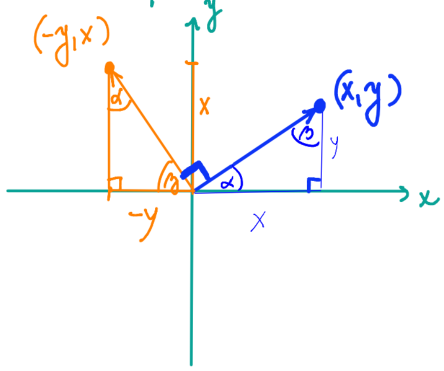
- $\dim(\operatorname{im} T) = \underline{3} \Rightarrow \dim(\ker T) = 3 - 3 = 0$
  $\Rightarrow \ker T = \{0\}$
- $\dim(\operatorname{im} T) = 2 \Rightarrow \dim(\ker T) = 3 - 2 = 1$
  $\Rightarrow \ker T = \mathcal{L}\{\vec{a}\}$
- $\dim(\operatorname{im} T) = 1 \Rightarrow \dim(\ker T) = 2$
- $\dim(\operatorname{im} T) = 0 \Rightarrow T(\vec{x}) = \vec{0}$
  $\Rightarrow T$ ničelna
  $\Rightarrow \dim(\ker T) = 3$

Nov pogled:

$A \in \mathbb{R}^{m \times n}$	$\varphi: \mathbb{R}^n \to \mathbb{R}^m$, $\vec{x} \mapsto A\vec{x}$
$N(A)$	$\ker \varphi$
$C(A)$	$\operatorname{im} \varphi$
A obrnljiva, $A \in \mathbb{R}^{n \times n}$	$\varphi: \mathbb{R}^n \to \mathbb{R}^n$, $\vec{x} \mapsto A\vec{x}$ bijekcija $\varphi^{-1}: \mathbb{R}^n \to \mathbb{R}^n$, $\vec{y} \mapsto A^{-1}\vec{y}$ inverzna preslikava

$A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$ $\rightsquigarrow$ $\varphi: \mathbb{R}^2\to\mathbb{R}^2$

$\begin{bmatrix}x\\y\end{bmatrix}\mapsto\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-y\\x\end{bmatrix}$ zasuk za $\frac{\pi}{2}$ okoli izhodišča



Kaj pa kompozitumi linearnih preslikav?

U,V,W vekt. prostori

$\left.\begin{aligned}\varphi: U\to V\\ \psi: V\to W\end{aligned}\right\}$ linearni preslikavi

$U \xrightarrow[A]{\varphi} V \xrightarrow[B]{\psi} W$

→ kompozitum: $(\psi\circ\varphi)(u)=\psi(\varphi(u))$ za vsak $u\in U$

$\psi\circ\varphi: U\to W$

$U=\mathbb{R}^n,\ V=\mathbb{R}^m,\ W=\mathbb{R}^p$

$A\in\mathbb{R}^{m\times n},\ B\in\mathbb{R}^{p\times m}$

⇒ matrika, ki pripada $\psi\circ\varphi$ je produkt matrik BA.

→ kompozitum asociativen ⇒ produkt matrik asociativen

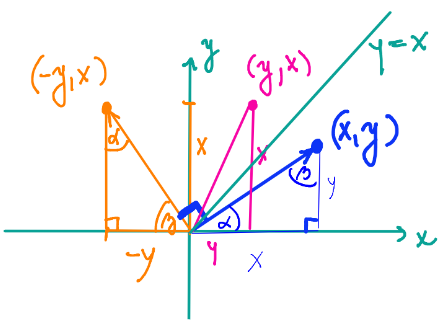
Zapišimo matriko, ki ustreza kompozitumu preslikav

- zasuka za $\frac{\pi}{2}$ okoli izhodišča ... $\mathcal{R}: \mathbb{R}^2\to\mathbb{R}^2$
- zrcaljenja čez y-os. ... $\mathcal{Z}: \mathbb{R}^2\to\mathbb{R}^2$

$\mathcal{Z}\circ\mathcal{R}=?$

preslikava	matrika
$\mathcal{R}$	$\begin{bmatrix}0&-1\\1&0\end{bmatrix}$
$\mathcal{Z}$	$\begin{bmatrix}-1&0\\0&1\end{bmatrix}$
$\varphi=\mathcal{Z}\circ\mathcal{R}$	$\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ ← matrika, ki pripada φ

$\varphi: \begin{bmatrix}x\\y\end{bmatrix}\mapsto\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}y\\x\end{bmatrix}$



$(x,y)\rightsquigarrow(y,x)$ preslikava: zrcaljenje čez $y=x$