Poglavje 1

DFT - Diskretna Fourierjeva transformacija

Naloga 1

Zapišite 4x4 ($n = 4$) DFT matriko v $\mathbb{C}$ in $\mathbb{Z}_5$.

Odgovor 1

V $\mathbb{C}$:

$$F = (\omega^{ij}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}, F^{-1} = (\omega^{-ij}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

V $\mathbb{Z}_5$:

$$F = (\omega^{ij}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & 4 & 1 & 4 \\ 1 & 3 & 4 & 2 \end{bmatrix}, F^{-1} = (\omega^{-ij}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 2 \\ 1 & 4 & 1 & 4 \\ 1 & 2 & 4 & 3 \end{bmatrix}$$

Naloga 2

Naredite transformacijo z DFT matriko v $\mathbb{C}$ in $\mathbb{Z}$ za naslednji polinom:

$$p(x) = 3 + x + 2x^2$$

Odgovor 2

$$a = [3, 1, 2, 0]^T$$

$$Fa = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 1+i \\ 4 \\ 1-i \end{bmatrix}$$

in

$$Fa = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & 4 & 1 & 4 \\ 1 & 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

Naloga 3

Zmnožite naslednja dva polinoma v $\mathbb{C}$ s pomočjo DFT ter inverzne DFT transformacije.

$$p(x) = 3 + x + 2x^2$$

$$q(x) = 1 + 2x$$

Odgovor 3

$$a = [3, 1, 2, 0]^T \text{ in } b = [1, 2, 0, 0]$$

$$Fa = [6, 1 + i, 4, 1 - i]^T$$

$$Fb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 + 2i \\ -1 \\ 1 - 2i \end{bmatrix}$$

$$c' = Fa \cdot Fb = [18, -1 + 3i, -4, -1 - 3i]$$

$$\frac{1}{4}F^{-1}c' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 18 \\ -1 + 3i \\ -4 \\ -1 - 3i \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 \\ 28 \\ 16 \\ 16 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 4 \\ 4 \end{bmatrix}$$

Naloga 4

Poiščimo vse PKE v $\mathbb{Z}_1 1$.

Odgovor 4

Naredimo naslednjo tabelo::

n	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	1	4	9	5	3	3	5	9	4	1
3		8	5	9	4	7	2	6	3	
4		5	4	3	9	9	3	4	5	
5		10	1	1	1	10	10	10	1	
6		9				5	4	3		
7		7				8	6	2		
8		3				4	9	5		
9		6				2	8	7		
10		1				1	1	1		

Iz katere lahko razberemo vse potence. Torej

- 1-PKE: 1
- 2-PKE: 10
- 5-PKE: 3,4 in 5,9. Tukaj opozorimo, da sta npr. 3 in 4 par PKE.
- 10-PKE: 2,6 in 7,8.