Assignment 3

Solve the following three exercises. Each exercise is worth five points. Solutions must be submitted by 5.5.2024. Use the link on e-ucilnica to turn in your work. The submission must be in pdf format.

Exercise 1: Amortization

You are working on an algorithm that adds rows and columns to a matrix. Each call to an *add()* function costs $i + c$ where i is the i-th call of the *add()* function, and c is some constant. Every call where $i = k^2$ for some k costs i^2. Meaning the cost function is :

$$c_i = \begin{cases} i + c & ; \quad i \neq k^2, k \in \mathbb{N} \\ i^2 & ; \quad i = k^2, k \in \mathbb{N} \end{cases}$$

what is the amortized cost of *add()* function?

Exercise 2: Amortization

You are developing a dynamic table that will only support insertions. Rather than doubling table size of the table when it becomes full, you decide to increase it by only 10%. Is the amortized cost of the insert still constant? Prove using the potential method.

Exercise 3: Approximation

Suppose you are working with a symmetric 4-SAT formula, described by 4-CNF formula F with n clauses, where each clause consists of 4 literals. For example: $F = (x_1 \vee x_2 \vee \neg x_4 \vee x_5) \wedge (x_4 \vee \neg x_2 \vee \neg x_1 \vee x_3) \wedge (\neg x_3 \vee x_2 \vee \neg x_5 \vee x_1)$.

In 4-SAT, we accept each clause if it evaluates to 1. In symmetric 4-SAT, we accept a clause if it evaluates to 1 and the clause in which we negate each literal also evaluates to 1. In other words, symmetric 4-SAT accepts each clause if it has one literal that assigns to 0 and one that assigns to 1.

A symmetric MAX 4-SAT is an NP-complete problem where we try to satisfy as many clauses as possible. We will use a simple approximation algorithm to solve the problem by setting each variable to 0 with a probability of 0.5 and to 1 with probability of 0.5.

Your task is to find the approximation factor for this algorithm.

Note: In 4-CNF, each clause can not have the same literal twice or have a variable x_i and its negation $\neg x_i$.