

5.) Janko opazi, da je čez noč zasnežilo dovoz do njegove hiše. Ker se mu mudi na avtobus, ne more skidati; na srečo pa je cesta splužena. Pod kolikšnim kotom glede na najkrajšo pod do ceste mora iti proti avtobusni postaji, če je Janko po dovozu od ceste oddaljen $l=750\text{m}$ in vstop na dovoz od avtobusne postaje $d=2500\text{m}$? Upoštevaj, da Janko lahko po snegu hodu s hitrostjo $v_1 = 1\text{m/s}$ in po spluženi cesti s hitrostjo $v_2 = 2\text{m/s}$. Dovoz je poljubno širok.

uberemo sledečo pot

$$S = S_1 + S_2$$

2

$$S = v_1 \cdot t_1 + v_2 \cdot t_2$$

čas hoje
po snegu

čas hoje
po cesti

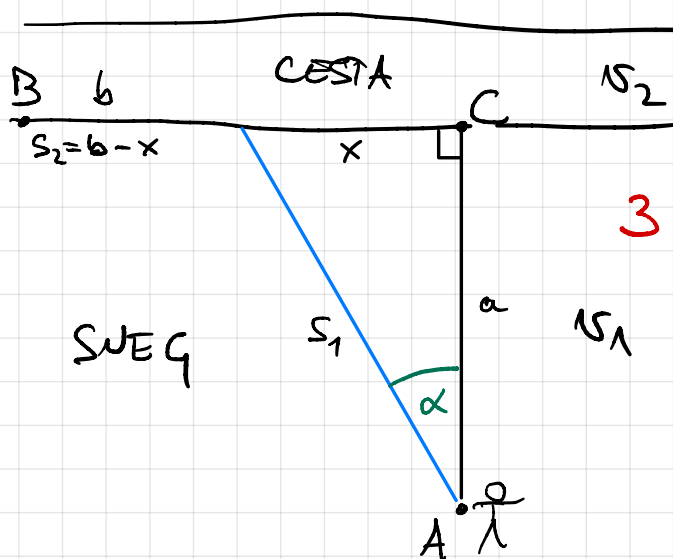
čas hoje $T = t_1 + t_2$ 1

$$t_1 = \frac{S_1}{v_1} \quad 2$$

$$S_1 = \sqrt{a^2 + x^2}$$

$$t_2 = \frac{S_2}{v_2} \quad 2$$

$$S_2 = b - x$$



$$T = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{b - x}{v_2} \Rightarrow T = T(x)$$

TO BO EKSTREM oz. MINIMUM $T(x)$ 5

$$0 = \frac{dT(x)}{dx} = T'(x) = \frac{d}{dx} \left\{ \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{b - x}{v_2} \right\}$$

$$T'(x) = \frac{1}{v_1} \cdot \frac{2x}{2\sqrt{a^2 + x^2}} - \frac{1}{v_2}$$

$$t'(x) = \frac{1}{v_1} \frac{x}{\sqrt{a^2 + x^2}} - \frac{1}{v_2} = 0 \quad 3$$

zdaj pa rešujemo enačbo:

$$\frac{1}{v_1} \frac{x}{\sqrt{a^2 + x^2}} = \frac{1}{v_2}$$

$$\frac{x}{\sqrt{a^2 + x^2}} = \frac{v_1}{v_2} \quad |^2$$

$$\frac{x^2}{a^2 + x^2} = \left(\frac{v_1}{v_2}\right)^2 \Rightarrow x^2 = \left(\frac{v_1}{v_2}\right)^2 a^2 + \left(\frac{v_1}{v_2}\right)^2 x^2$$

$$x^2 \left(1 - \left(\frac{v_1}{v_2}\right)^2\right) = \left(\frac{v_1}{v_2}\right)^2 a^2$$

$$x^2 = \frac{\left(\frac{v_1}{v_2}\right)^2 a^2}{\left(1 - \left(\frac{v_1}{v_2}\right)^2\right)} = \frac{a^2}{\left(\frac{v_2}{v_1}\right)^2 - 1}$$

$$x = \pm \frac{a}{\sqrt{\left(\frac{v_2}{v_1}\right)^2 - 1}} \quad 2$$

$$\begin{aligned} \operatorname{tg} \alpha &= \frac{x}{a} = \frac{1}{\sqrt{\left(\frac{v_2}{v_1}\right)^2 - 1}} = \frac{1}{\sqrt{\left(\frac{3}{1}\right)^2 - 1}} \\ &= \frac{1}{\sqrt{4-1}} = \frac{1}{\sqrt{3}} \end{aligned} \quad 5$$

$$\operatorname{tg} \alpha = \frac{1}{\sqrt{3}} \Rightarrow \underline{\underline{\alpha = 30^\circ}}$$