

Naloga: 1.) S preverjanjem pripadnosti zaprtim razredom pokaži, da je nabor  $\{v, \neg\}$  funkcijsko poln. Pokaži še, da nabor  $\{v, \cdot\}$  ni funkcijsko poln.

$$\{v, \neg\}: T_0: v: f(0,0) = 0 \vee 0 = 0; \quad v \in T_0$$

$$\neg: f(0) = \bar{0} = 1; \quad \neg \notin T_0$$

$$T_1: v: f(1,1) = 1 \vee 1 = 1; \quad v \in T_1$$

$$\neg: f(1) = \bar{1} = 0; \quad \neg \notin T_1$$

Si v:

| $x_1$ | $x_2$ | $v$ |
|-------|-------|-----|
| 0     | 0     | 0   |
| 0     | 1     | 1   |
| 1     | 0     | 1   |
| 1     | 1     | 1   |

$$f(\vec{w}_1) = f(\vec{w}_2); \quad v \notin S$$

$$\neg: x \mid \neg$$

|   |   |
|---|---|
| 0 | 1 |
| 1 | 0 |

$$f(\vec{w}_2) \neq f(\vec{w}_1); \quad \neg \in S$$

$$L: v: \begin{matrix} x_1 \\ x_2 \end{matrix}$$

|   |   |
|---|---|
| 1 | 1 |
| 1 | 0 |

$$\bar{x}_1 \bar{x}_2: \bar{x}_1 x_2 \text{ popolnoma različna}$$

$$x_1: \bar{x}_1 \text{ niti popolnoma enake, niti}$$

$$\text{popolnoma različna; } v \notin L$$

$$\neg: \begin{matrix} x_1 \\ x_2 \end{matrix}$$

|   |  |
|---|--|
| 1 |  |
|---|--|

$$x: \bar{x} \text{ popolnoma različna; } \neg \in L$$

$$M: v: \begin{matrix} x_1 & x_2 & v \end{matrix}$$

$$0 \ 0 \ 0$$

$$\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$$

$$0 \ 1 \ 1$$

$$\vec{w}_0 < \vec{w}_2; \quad f(\vec{w}_0) \leq f(\vec{w}_2)$$

$$1 \ 0 \ 1$$

$$\vec{w}_1 < \vec{w}_3; \quad f(\vec{w}_1) \leq f(\vec{w}_3)$$

$$1 \ 1 \ 1$$

$$\vec{w}_2 < \vec{w}_3; \quad f(\vec{w}_2) \leq f(\vec{w}_3); \quad v \in M$$

$$\neg: x \mid \neg$$

$$0 \ 1$$

$$\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) > f(\vec{w}_1); \quad \neg \notin M$$

$$1 \ 0$$

7 odpira razrede  $T_0, T_1, M$ ,  $v$  pa odpira razreda  $S$  in  $L$ .  
nabor  $\{v, \neg\}$  je FPS

$$\{v_i\}; T_0: v; f(0,0)=0 \vee 0=0; \quad v \in T_0$$

$$\therefore f(0,0)=0 \cdot 0=0; \quad \cdot \in T_0$$

$$T_1: v; f(1,1)=1 \vee 1=1; \quad v \in T_1$$

$$\therefore f(1,1)=1 \cdot 1=1; \quad \cdot \in T_1$$

$$S: v; \overline{x_1} \vee \overline{x_2} = x_1 \cdot x_2 \neq x_1 \vee x_2; \quad v \notin S$$

$$\therefore \overline{x_1} \cdot \overline{x_2} = x_1 \vee x_2 \neq x_1 x_2; \quad \cdot \notin S$$

| $L: v; x_1 x_2$ | $v$ | $f(x_1, x_2)$ | $f(x_1, x_2) = a_0 \vee a_1 x_1 \vee a_2 x_2$             |
|-----------------|-----|---------------|-----------------------------------------------------------|
| 0 0             | 0   | 0             | $f(0,0) = a_0 \vee a_1 0 \vee a_2 0 = a_0; \quad a_0 = 0$ |
| 0 1             | 1   | 1             | $f(0,1) = 0 \vee a_1 0 \vee a_2 1 = a_2; \quad a_2 = 1$   |
| 1 0             | 1   | 1             | $f(1,0) = 0 \vee a_1 1 \vee a_2 0 = a_1; \quad a_1 = 1$   |
| 1 1             | 1   | 0             |                                                           |

$$f(x_1, x_2) = 0 \vee 1 x_1 \vee 1 x_2 =$$

$$= x_1 \vee x_2 =$$

$$= x_1 \overline{x_2} \vee \overline{x_1} x_2; \quad v \notin L$$

| $\therefore x_1 x_2$ | $\cdot$ | $f(x_1, x_2)$ | $f(x_1, x_2) = a_0 \vee a_1 x_1 \vee a_2 x_2$             |
|----------------------|---------|---------------|-----------------------------------------------------------|
| 0 0                  | 0       | 0             | $f(0,0) = a_0 \vee a_1 0 \vee a_2 0 = a_0; \quad a_0 = 0$ |
| 0 1                  | 0       | 0             | $f(0,1) = 0 \vee a_1 0 \vee a_2 1 = a_2; \quad a_2 = 0$   |
| 1 0                  | 0       | 0             | $f(1,0) = 0 \vee a_1 1 \vee a_2 0 = a_1; \quad a_1 = 0$   |
| 1 1                  | 1       | 0             |                                                           |

$$f(x_1, x_2) = 0 \vee 0 x_1 \vee 0 x_2 = 0; \quad \cdot \notin L$$

| $M: v; x_1 x_2$ | $v$ | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                |
|-----------------|-----|------------------------------------------------------------------------------|
| 0 0             | 0   | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                |
| 0 1             | 1   | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                |
| 1 0             | 1   | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                |
| 1 1             | 1   | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1); \quad v \in M$ |

| $\therefore x_1 x_2$ | $\cdot$ | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                    |
|----------------------|---------|----------------------------------------------------------------------------------|
| 0 0                  | 0       | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                    |
| 0 1                  | 0       | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                    |
| 1 0                  | 0       | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1)$                    |
| 1 1                  | 1       | $\vec{w}_0 < \vec{w}_1; \quad f(\vec{w}_0) \leq f(\vec{w}_1); \quad \cdot \in M$ |

$V$  in: pripadati razredom  $T_0, T_1$  in  $M$  in jih ne odpirata.  $\Rightarrow$  ni FPS

2.) Določi pripadnost funkcije  $f$  zaprtim razredom in jo realiziraj z operacijami AND, OR, NEG. Linearnost preveri analitično in s Karnaughjevim diagramom. Po potrebi funkcijo dopolni do funkcijske popolnosti!

$$f(x_1, x_2, x_3, x_4) = (x_1 \rightarrow x_2) \nabla (x_3 \equiv x_4)$$

$$\begin{aligned} f(x_1, x_2, x_3, x_4) &= (x_1 \rightarrow x_2) \nabla (x_3 \equiv x_4) = \\ &= (\bar{x}_1 \vee x_2) \nabla (x_3 x_4 \vee \bar{x}_3 \bar{x}_4) = \\ &= (\bar{x}_1 \vee x_2) (x_3 x_4 \vee \bar{x}_3 \bar{x}_4) \vee (\bar{x}_1 \vee x_2) (\bar{x}_3 x_4 \vee \bar{x}_3 \bar{x}_4) = \\ &= x_1 \bar{x}_2 (x_3 x_4 \vee \bar{x}_3 \bar{x}_4) \vee (\bar{x}_1 \vee x_2) (\bar{x}_3 \vee \bar{x}_4) (x_3 \vee x_4) = \\ &= x_1 \bar{x}_2 x_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 \bar{x}_3 x_4 \vee \bar{x}_1 x_3 \bar{x}_4 \vee x_2 \bar{x}_3 x_4 \vee x_2 x_3 \bar{x}_4 = \\ &= x_1 \bar{x}_2 x_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 \vee \bar{x}_1 x_2 \bar{x}_3 x_4 \vee \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 \vee \bar{x}_1 x_2 x_3 x_4 \vee \\ &\quad x_1 x_2 \bar{x}_3 x_4 \vee x_1 x_2 x_3 \bar{x}_4 = \\ &= \vee^4(1, 2, 5, 6, 8, 11, 13, 14) \end{aligned}$$

$$T_0: f(0,0,0,0) = (0 \rightarrow 0) \nabla (0 \equiv 0) = 1 \nabla 1 = 0; \quad f \in T_0$$

$$T_1: f(1,1,1,1) = (1 \rightarrow 1) \nabla (1 \equiv 1) = 1 \nabla 1 = 0; \quad f \notin T_1$$

| $S_i$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $f$ |
|-------|-------|-------|-------|-------|-----|
|       | 0     | 0     | 0     | 0     | 0   |
|       | 0     | 0     | 0     | 1     | 1   |
|       | 0     | 0     | 1     | 0     | 1   |
|       | 0     | 0     | 1     | 1     | 0   |
|       | 0     | 1     | 0     | 0     | 0   |
|       | 0     | 1     | 0     | 1     | 1   |
|       | 0     | 1     | 1     | 0     | 1   |
|       | 0     | 1     | 1     | 1     | 0   |
|       | 1     | 0     | 0     | 0     | 1   |
|       | 1     | 0     | 0     | 1     | 0   |
|       | 1     | 0     | 1     | 0     | 0   |
|       | 1     | 0     | 1     | 1     | 1   |
|       | 1     | 1     | 0     | 0     | 0   |
|       | 1     | 1     | 0     | 1     | 1   |
|       | 1     | 1     | 1     | 0     | 1   |
|       | 1     | 1     | 1     | 1     | 0   |

$$f(\vec{w}_3) = f(\vec{w}_m); \quad f \notin S$$

|           |    |           |    |    |    |
|-----------|----|-----------|----|----|----|
|           |    | $x_3 x_4$ |    |    |    |
|           |    | 00        | 01 | 11 | 10 |
| $x_1 x_2$ | 00 |           | 1  |    | 1  |
|           | 01 |           | 1  |    | 1  |
|           | 11 |           | 1  |    | 1  |
|           | 10 | 1         |    | 1  |    |

$x_1 x_3 x_4$ ;  $x_1 x_3 \bar{x}_4$  POGLNOMA RAZLIČNA

$\bar{x}_2 x_3$ ;  $x_1 x_2$  NITI POGLNOMA ENAKA,  
NITI POGLNOMA RAZLIČNA

↓  
 $f \notin L$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $f$ | $f_c$ | $f_c = a_0 \nabla a_1 x_1 \nabla a_2 x_2 \nabla a_3 x_3 \nabla a_4 x_4$ |
|-------|-------|-------|-------|-----|-------|-------------------------------------------------------------------------|
| 0     | 0     | 0     | 0     | 0   | 0     | $f(0,0,0,0) = a_0 \nabla 0 \nabla 0 \nabla 0 \nabla 0 = a_0$            |
| 0     | 0     | 0     | 1     | 1   | 1     | $f(0,0,0,1) = 0 \nabla 0 \nabla 0 \nabla 0 \nabla a_4 = a_4$            |
| 0     | 0     | 1     | 0     | 1   | 1     | $f(0,0,1,0) = 0 \nabla 0 \nabla 0 \nabla a_3 \nabla 0 = a_3$            |
| 0     | 0     | 1     | 1     | 0   | 0     | $f(0,0,1,1) = 0 \nabla 0 \nabla a_3 \nabla 0 \nabla 0 = a_3$            |
| 0     | 1     | 0     | 0     | 0   | 0     | $f(0,1,0,0) = 0 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_2$            |
| 0     | 1     | 0     | 1     | 1   | 1     | $f(0,1,0,1) = 0 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_2$            |
| 0     | 1     | 1     | 0     | 1   | 1     | $f(0,1,1,0) = 0 \nabla a_2 \nabla 0 \nabla a_3 \nabla 0 = a_2$          |
| 0     | 1     | 1     | 1     | 0   | 0     | $f(0,1,1,1) = 0 \nabla a_2 \nabla a_3 \nabla 0 \nabla 0 = a_2$          |
| 1     | 0     | 0     | 0     | 1   | 1     | $f(1,0,0,0) = a_1 \nabla 0 \nabla 0 \nabla 0 \nabla 0 = a_1$            |
| 1     | 0     | 0     | 1     | 0   | 0     | $f(1,0,0,1) = a_1 \nabla 0 \nabla 0 \nabla 0 \nabla a_4 = a_1$          |
| 1     | 0     | 1     | 0     | 0   | 0     | $f(1,0,1,0) = a_1 \nabla 0 \nabla 0 \nabla a_3 \nabla 0 = a_1$          |
| 1     | 0     | 1     | 1     | 1   | 1     | $f(1,0,1,1) = a_1 \nabla 0 \nabla 0 \nabla a_3 \nabla a_4 = a_1$        |
| 1     | 1     | 0     | 0     | 0   | 0     | $f(1,1,0,0) = a_1 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_1$          |
| 1     | 1     | 0     | 1     | 1   | 1     | $f(1,1,0,1) = a_1 \nabla a_2 \nabla 0 \nabla 0 \nabla a_4 = a_1$        |
| 1     | 1     | 1     | 0     | 1   | 1     | $f(1,1,1,0) = a_1 \nabla a_2 \nabla 0 \nabla a_3 \nabla 0 = a_1$        |
| 1     | 1     | 1     | 1     | 0   | 0     | $f(1,1,1,1) = a_1 \nabla a_2 \nabla a_3 \nabla 0 \nabla 0 = a_1$        |

$f_c = a_0 \nabla a_1 x_1 \nabla a_2 x_2 \nabla a_3 x_3 \nabla a_4 x_4$

$f(0,0,0,0) = a_0 \nabla 0 \nabla 0 \nabla 0 \nabla 0 = a_0$   $a_0 = 0$

$f(0,0,0,1) = 0 \nabla 0 \nabla 0 \nabla 0 \nabla a_4 = a_4$   $a_4 = 1$

$f(0,0,1,0) = 0 \nabla 0 \nabla 0 \nabla a_3 \nabla 0 = a_3$   $a_3 = 1$

$f(0,0,1,1) = 0 \nabla 0 \nabla a_3 \nabla 0 \nabla 0 = a_3$   $a_3 = 0$

$f(0,1,0,0) = 0 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_2$   $a_2 = 1$

$f(0,1,0,1) = 0 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_2$   $a_2 = 1$

$f(0,1,1,0) = 0 \nabla a_2 \nabla 0 \nabla a_3 \nabla 0 = a_2$   $a_2 = 1$

$f(0,1,1,1) = 0 \nabla a_2 \nabla a_3 \nabla 0 \nabla 0 = a_2$   $a_2 = 1$

$f(1,0,0,0) = a_1 \nabla 0 \nabla 0 \nabla 0 \nabla 0 = a_1$   $a_1 = 1$

$f(1,0,0,1) = a_1 \nabla 0 \nabla 0 \nabla 0 \nabla a_4 = a_1$   $a_1 = 1$

$f(1,0,1,0) = a_1 \nabla 0 \nabla 0 \nabla a_3 \nabla 0 = a_1$   $a_1 = 1$

$f(1,0,1,1) = a_1 \nabla 0 \nabla 0 \nabla a_3 \nabla a_4 = a_1$   $a_1 = 1$

$f(1,1,0,0) = a_1 \nabla a_2 \nabla 0 \nabla 0 \nabla 0 = a_1$   $a_1 = 1$

$f(1,1,0,1) = a_1 \nabla a_2 \nabla 0 \nabla 0 \nabla a_4 = a_1$   $a_1 = 1$

$f(1,1,1,0) = a_1 \nabla a_2 \nabla 0 \nabla a_3 \nabla 0 = a_1$   $a_1 = 1$

$f(1,1,1,1) = a_1 \nabla a_2 \nabla a_3 \nabla 0 \nabla 0 = a_1$   $a_1 = 1$

M:  $\vec{w}_0 < \vec{w}_1$ ;  $f(\vec{w}_0) \leq f(\vec{w}_1)$

$\vec{w}_0 < \vec{w}_2$ ;  $f(\vec{w}_0) \leq f(\vec{w}_2)$

$\vec{w}_0 < \vec{w}_3$ ;  $f(\vec{w}_0) \leq f(\vec{w}_3)$

$\vec{w}_0 < \vec{w}_4$ ;  $f(\vec{w}_0) \leq f(\vec{w}_4)$

$\vec{w}_1 < \vec{w}_2$ ;  $f(\vec{w}_1) > f(\vec{w}_2)$

$f \notin M$