Naloga: 1.) S preverjanjem pripadnosti zaprtim razredom pokaži, da je nabor $\{\vee, \neg\}$ funkcijsko poln. Pokaži še, da nabor $\{\vee, \cdot\}$ ni funkcijsko poln.

$\{\vee, \neg\}$: T_0: $\vee$: $f(0,0) = 0 \vee 0 = 0$; $\quad \vee \in T_0$

$\neg$: $f(0) = \bar{0} = 1$; $\quad \neg \notin T_0$

T_1: $\vee$: $f(1,1) = 1 \vee 1 = 1$; $\quad \vee \in T_1$

$\neg$: $f(1) = \bar{1} = 0$; $\quad \neg \notin T_1$

S: $\vee$:

x_1	x_2	$\vee$
0	0	0
0	1	1
1	0	1
1	1	1

$f(\vec{w}_1) = f(\vec{w}_2)$; $\quad \vee \notin S$

$\neg$:

x	$\neg$
0	1
1	0

$f(\vec{w}_0) \neq f(\vec{w}_1)$; $\quad \neg \in S$

L: $\vee$:

	x_1	
x_2	1	1
	1	

$\bar{x}_1\bar{x}_2 : \bar{x}_1 x_2$ popolnoma različna

$x_1 : \bar{x}_1$ niti popolnoma enaka, niti popolnoma različna; $\quad \vee \notin L$

$\neg$:

x	
1	

$x : \bar{x}$ popolnoma različna; $\quad \neg \in L$

M: $\vee$:

x_1	x_2	$\vee$
0	0	0
0	1	1
1	0	1
1	1	1

$\vec{w}_0 < \vec{w}_1$; $f(\vec{w}_0) \leq f(\vec{w}_1)$

$\vec{w}_0 < \vec{w}_2$; $f(\vec{w}_0) \leq f(\vec{w}_2)$

$\vec{w}_1 < \vec{w}_3$; $f(\vec{w}_1) \leq f(\vec{w}_3)$

$\vec{w}_2 < \vec{w}_3$ $f(\vec{w}_2) \leq f(\vec{w}_3)$; $\quad \vee \in M$

$\neg$:

x	$\neg$
0	1
1	0

$\vec{w}_0 < \vec{w}_1$; $f(\vec{w}_0) > f(\vec{w}_1)$; $\quad \neg \notin M$

$\neg$ odpira razrede T_0, T_1, M, $\vee$ pa odpira razreda S in L.

$\Downarrow$

nabor $\{\vee, \neg\}$ je FPS

$\{\vee, \cdot\}$: T_0: $\vee$: $f(0,0) = 0 \vee 0 = 0$; $\quad \vee \in T_0$

$\cdot$: $f(0,0) = 0 \cdot 0 = 0$; $\quad \cdot \in T_0$

T_1: $\vee$: $f(1,1) = 1 \vee 1 = 1$; $\quad \vee \in T_1$

$\cdot$: $f(1,1) = 1 \cdot 1 = 1$; $\quad \cdot \in T_1$

S: $\vee$: $\overline{\bar{x}_1 \vee \bar{x}_2} = x_1 \cdot x_2 \neq x_1 \vee x_2$; $\quad \vee \notin S$

$\cdot$: $\overline{\bar{x}_1 \cdot \bar{x}_2} = x_1 \vee x_2 \neq x_1 x_2$; $\quad \cdot \notin S$

L: $\vee$:

x_1 x_2	$\vee$	$f(x_1,x_2)_L$
0 0	0	0
0 1	1	1
1 0	1	1
1 1	1 $\neq$	0

$f(x_1,x_2)_L = a_0 \nabla a_1 x_1 \nabla a_2 x_2$

$f(0,0) = a_0 \nabla a_1 0 \nabla a_2 0 = a_0$; $\quad a_0 = 0$

$f(0,1) = 0 \nabla a_1 0 \nabla a_2 1 = a_2$; $\quad a_2 = 1$

$f(1,0) = 0 \nabla a_1 1 \nabla a_2 0 = a_1$; $\quad a_1 = 1$

$f(x_1,x_2) = 0 \nabla 1x_1 \nabla 1x_2 =$
$= x_1 \nabla x_2 =$
$= x_1 \bar{x}_2 \vee \bar{x}_1 x_2$; $\quad \vee \notin L$

$\cdot$:

x_1 x_2	$\cdot$	$f(x_1,x_2)_L$
0 0	0	0
0 1	0	0
1 0	0	0
1 1	1 $\neq$	0

$f(x_1,x_2)_L = a_0 \nabla a_1 x_1 \nabla a_2 x_2$

$f(0,0) = a_0 \nabla a_1 0 \nabla a_2 0 = a_0$ $\quad a_0 = 0$

$f(0,1) = 0 \nabla a_1 0 \nabla a_2 1 = a_2$ $\quad a_2 = 0$

$f(1,0) = 0 \nabla a_1 1 \nabla a_2 0 = a_1$ $\quad a_1 = 0$

$f(x_1,x_2)_L = 0 \nabla 0x_1 \nabla 0x_2 = 0$; $\quad \cdot \notin L$

M: $\vee$:

x_1 x_2	$\vee$		
0 0	0	$\vec{\omega}_0 < \vec{\omega}_1$;	$f(\vec{\omega}_0) \leq f(\vec{\omega}_1)$
0 1	1	$\omega_0 < \vec{\omega}_2$;	$f(\vec{\omega}_0) \leq f(\vec{\omega}_2)$
1 0	1	$\vec{\omega}_1 < \vec{\omega}_3$;	$f(\vec{\omega}_1) \leq f(\vec{\omega}_3)$
1 1	1	$\vec{\omega}_2 < \vec{\omega}_3$;	$f(\vec{\omega}_2) \leq f(\vec{\omega}_3)$; $\quad \vee \in M$

$\cdot$:

x_1 x_2	$\cdot$		
0 0	0	$\vec{\omega}_0 < \vec{\omega}_1$;	$f(\vec{\omega}_0) \leq f(\vec{\omega}_1)$
0 1	0	$\vec{\omega}_0 < \vec{\omega}_2$;	$f(\vec{\omega}_0) \leq f(\vec{\omega}_2)$
1 0	0	$\vec{\omega}_1 < \vec{\omega}_3$;	$f(\vec{\omega}_1) \leq f(\vec{\omega}_3)$
1 1	1	$\vec{\omega}_2 < \vec{\omega}_3$;	$f(\vec{\omega}_2) \leq f(\vec{\omega}_3)$; $\quad \cdot \in M$

$\vee$ in $\cdot$ pripadata razredom T_0, T_1 in M in jih ne odpirata $\Rightarrow$ ni FPS

2) Določi pripadnost funkcije f zaprtim razredom in jo realiziraj z operacijami AND, OR, NEG. Linearnost preveri analitično in s Karnaughjevim diagramom. Po potrebi funkcijo dopolni do funkcijske popolnosti!

$$f(x_1, x_2, x_3, x_4) = (x_1 \rightarrow x_2) \nabla (x_3 \equiv x_4)$$

$$f(x_1, x_2, x_3, x_4) = (x_1 \rightarrow x_2) \nabla (x_3 \equiv x_4) =$$

$$= (\bar{x}_1 \vee x_2) \nabla (x_3 x_4 \vee \bar{x}_3 \bar{x}_4) =$$

$$= (\overline{\bar{x}_1 \vee x_2})(x_3 x_4 \vee \bar{x}_3 \bar{x}_4) \vee (\bar{x}_1 \vee x_2)(\overline{x_3 x_4 \vee \bar{x}_3 \bar{x}_4}) =$$

$$= x_1 \cdot \bar{x}_2 (x_3 x_4 \vee \bar{x}_3 \bar{x}_4) \vee (\bar{x}_1 \vee x_2)(\bar{x}_3 \vee \bar{x}_4) \cdot (x_3 \vee x_4) =$$

$$= \underset{11}{x_1 \bar{x}_2 x_3 x_4} \vee \underset{8}{x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4} \vee \underset{1}{\bar{x}_1 \bar{x}_3 x_4} \vee \underset{5}{\bar{x}_1 x_3 \bar{x}_4} \vee \underset{2}{x_2 \bar{x}_3 x_4} \vee \underset{6}{x_2 x_3 \bar{x}_4} =$$

$$= x_1 \bar{x}_2 x_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \underset{13}{\bar{x}_1 \bar{x}_2 \bar{x}_3 x_4} \vee \underset{14}{\bar{x}_1 x_2 \bar{x}_3 x_4} \vee \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 \vee \bar{x}_1 x_2 x_3 \bar{x}_4 \vee$$

$$x_1 x_2 \bar{x}_3 x_4 \vee x_1 x_2 x_3 \bar{x}_4 =$$

$$= \vee^4(1, 2, 5, 6, 8, 11, 13, 14)$$

T_0: $f(0,0,0,0) = (0 \rightarrow 0) \nabla (0 \equiv 0) = 1 \nabla 1 = 0$; $f \in T_0$

T_1: $f(1,1,1,1) = (1 \rightarrow 1) \nabla (1 \equiv 1) = 1 \nabla 1 = 0$; $f \notin T_1$

S:

x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$f(\vec{w}_3) = f(\vec{w}_{11})$; $f \notin S$

	x_3x_4 00	01	11	10
x_1x_2 00		1		1
01		1		1
11		1		1
10	1		1	

$x_1x_3x_4 : x_1x_3\bar{x}_4$ popolnoma različna

$\bar{x}_1x_3 : x_1x_3$ niti popolnoma enaka,

niti popolnoma različna

$\Downarrow$

$f \notin L$

x_1	x_2	x_3	x_4	f	f_L
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	1
1	1	0	0	0 ≠	1
1	1	0	1	1 ≠	0
1	1	1	0	1 ≠	0
1	1	1	1	0 ≠	1

$f_L = a_0 \nabla a_1x_1 \nabla a_2x_2 \nabla a_3x_3 \nabla a_4x_4$

$f(0,0,0,0)_L = a_0 \nabla 0 \nabla 0 \nabla 0 \nabla 0 = a_0$ $\qquad a_0 = 0$

$f(0,0,0,1)_L = 0 \nabla 0 \nabla 0 \nabla 0 \nabla a_4 = a_4$ $\qquad a_4 = 1$

$f(0,0,1,0)_L = 0 \nabla 0 \nabla 0 \nabla a_3 \nabla 0 = a_3$ $\qquad a_3 = 1$

$f(0,1,0,0)_L = 0 \nabla 0 \nabla a_2 \nabla 0 \nabla 0 = a_2$ $\qquad a_2 = 0$

$f(1,0,0,0)_L = 0 \nabla a_1 \nabla 0 \nabla 0 \nabla 0 = a_1$ $\qquad a_1 = 1$

$f(x_1,x_2,x_3,x_4)_L = 0 \nabla 1x_1 \nabla 0x_2 \nabla 1x_3 \nabla 1x_4 =$

$= x_1 \nabla x_3 \nabla x_4$

$f(\vec{w}_{12}) \neq f(\vec{w}_{12})_L$

$f(\vec{w}_{13}) \neq f(\vec{w}_{13})_L$

$f(\vec{w}_{15}) \neq f(\vec{w}_{15})_L$; $\qquad f \notin L$

$f(\vec{w}_{14}) \neq f_L(\vec{w}_{14})$

M: $\vec{w}_0 < \vec{w}_1$; $\quad f(\vec{w}_0) \leq f(\vec{w}_1)$

$\vec{w}_0 < \vec{w}_2$; $\quad f(\vec{w}_0) \leq f(\vec{w}_2)$

$\vec{w}_0 < \vec{w}_4$; $\quad f(\vec{w}_0) \leq f(\vec{w}_4)$

$\vec{w}_0 < \vec{w}_8$; $\quad f(\vec{w}_0) \leq f(\vec{w}_8)$

$\vec{w}_1 < \vec{w}_3$; $\quad f(\vec{w}_1) > f(\vec{w}_3)$; $\qquad f \notin M$

⋮