1.

a) $v = at + v_0^{=0} = 3\,m/s^2 \cdot 5\,s = 15\,m/s$ (5)

b) Prevožena pot do A:

$s_1 = \frac{at^2}{2} + v_0^{=0}t = \frac{3\,m/s^2 \cdot 5\,s^2}{2} = 37,5\,m$ (5)

Prevožena pot med A in B:

$s_2 = \frac{at^2}{2} + v_0 t = \frac{-2\,m/s^2\ 2^2\,s}{2} + 15\,m/s \cdot 2\,s = 26\,m$ (5) (5)

$s = s_1 + s_2 = 37,5\,m + 26\,m = 63,5\,m$

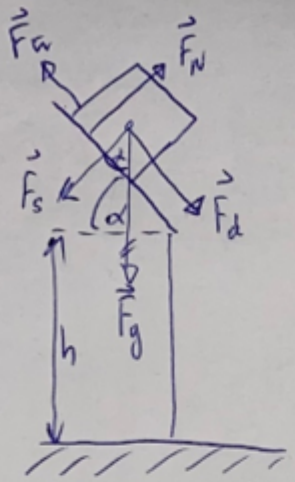
c) $a = 0$ (5)

$\alpha = 30°$

$k_{tr} = 0{,}3$

$s = 2\,m$

$h = 10\,m$



$\parallel$: $ma = F_d - F_{tr}$ (2)

$F_d = mg\sin\alpha$

$\perp$: $F_N = F_s$

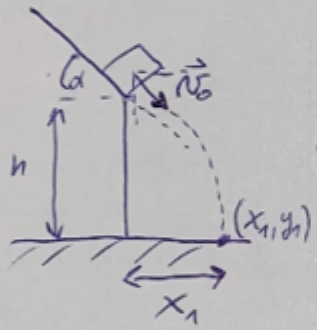
$F_s = mg\cos\alpha$

$F_{tr} = k_{tr} \cdot F_N$

(2) (5)

$ma = mg\sin\alpha - k_{tr} mg\cos\alpha$

$a = 2{,}356\ m/s^2$ (1)



$v_0 = \sqrt{2as} = 3{,}07\ m/s$ (5)

$v_{0x} = v_0\cos\alpha$ (1)

$v_{0y} = v_0\sin\alpha$

$x_1 = v_{0x} \cdot t_q$

$y_1 = h - v_{0y} t_q - \frac{g t_1^2}{2}$ (7)

$y_1 = 0$

$0 = -\parallel-$

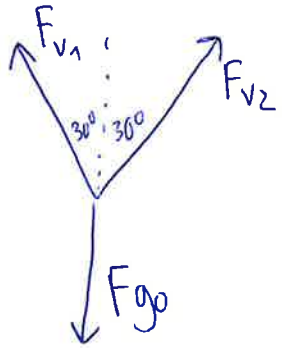
$$t_1 = \frac{-(-v_{0y}) \pm \sqrt{v_{0y}^2 - 4\cdot\left(-\frac{g}{2}\right)\cdot h}}{2\cdot\left(-\frac{g}{2}\right)}$$ (2)

$t_1 = \rightarrow 1{,}28\,s$

$\searrow -1{,}59\,s$ //

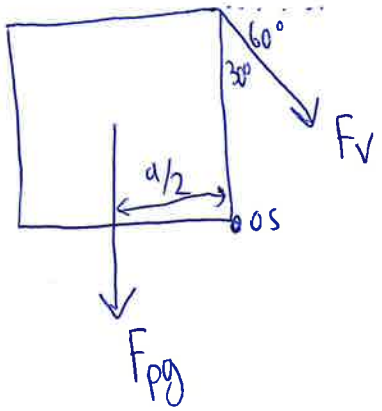
$x_1 = 3{,}40\,m$

3)



$$\cos 30^\circ = \frac{F_{go}/2}{F_V}$$

$$F_V = \frac{F_{go}}{2\cos 30^\circ} = \frac{m_o g}{2\cdot\cos 30^\circ} \quad (5)$$



$$M_{pg} = M_V \quad (5)$$

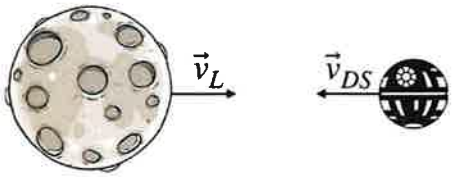
$$M_{pg} = m_p \cdot g \cdot \frac{a}{2} \quad (5)$$

$$M_V = F_V \cdot a \cdot \sin 30^\circ \quad (5)$$

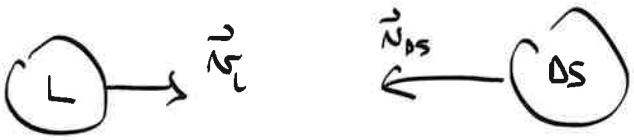
$$m_p g \frac{a}{2} = \frac{m_o g}{2\cos 30^\circ} a \sin 30^\circ$$

$$m_o = \frac{m_p}{\tan 30^\circ} = 15{,}59\,kg \quad (5)$$

Darth Vader se vozi po našem osončju v svojem Death Starju s hitrostjo $v_{DS} = 700{,}0\,\mathrm{km/s}$ po poti, ki bo direktno trčila z Luno, kot je prikazano na skici. Pri trku se Death Star zlepi z Luno, nas kot opazovalce pa zanimajo posledice. Kolikšna je hitrost skupka Luna - Death Star po trku? Za koliko se dolgo po trku podaljša Lunin mesec? Upoštevaj sledeče: masa Lune $m_L = 7{,}347 \times 10^{22}\,\mathrm{kg}$, masa Death Starja $m_{DS} = 6{,}817 \times 10^{17}\,\mathrm{kg}$, masa Zemlje $m_Z = 6{,}024 \times 10^{24}\,\mathrm{kg}$, Lunin mesec traja 27,30 dni ter Luna kroži okrog Zemlje pri radiju $R = 384\,000\,\mathrm{km}$.



a) Neprožni trk Death Star in Luna



TRK V 1D

→ ohranitev gibalne količine

$\vec{p}^{(z)} = \vec{p}^{(k)}$ pomemben predznak!

$m_L \vec{v}_L - m_{DS} \vec{v}_{DS} = (m_L + m_{DS})\,\vec{v}'$

v 1D: $m_L v_L - m_{DS} v_{DS} = (m_L + m_{DS})\,v'$

$$v' = \frac{m_L v_L - m_{DS} v_{DS}}{m_L + m_{DS}} \quad (4)$$

$v' = 1016{,}4\,\mathrm{m/s}$ (4)

$m_L = 7.347 \cdot 10^{22}\,\mathrm{kg}$

$m_{DS} = 6.817 \cdot 10^{17}\,\mathrm{kg}$

$v_{DS} = 700.0\,\mathrm{m/s}$

$m_Z = 6.024 \cdot 10^{24}\,\mathrm{kg}$

→ hitrost Lune

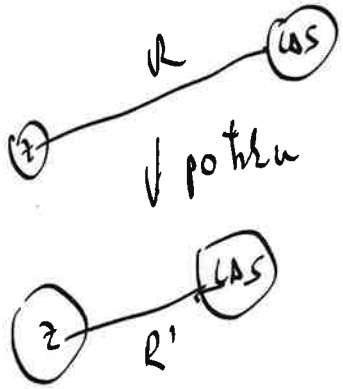
- kroži okrog zemlje pri $R = 384\,000\,\mathrm{km}$
- en krog naredi vsake $t_0 = 27.3\,\mathrm{dni}$

$v_L = \omega_L R = \frac{2\pi}{t_0} \cdot R = 1022.9\,\mathrm{m/s}$ (4)

→ spremenba orbite stopla LOS: v'

$$F_g = F_{cp}$$

$$\frac{G \cancel{m_{LOS}} m_z}{R'^2} = \frac{\cancel{m_{LOS}} v'^2}{R'} \quad (4)$$



$$\frac{G m_z}{R'} = v'^2$$

$$R' = \frac{G m_z}{v'^2} = 388340\ km \quad (4)$$

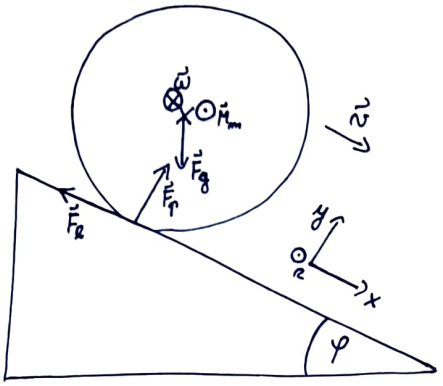
→ za koliko se spremeni čas meseca?

$$t_0' = \frac{2\pi R'}{v'} = 27{,}83\ dni$$

(4) + (1) → za $\Delta t = 0{,}53\ dni$

FRI 1. kolokvij 24/25 (11. 12. 2024)

5.



AVTO: $\sum \vec{F} = m\vec{a}$ $\quad N_s = 4$

x: $\quad mg\sin\varphi - N_s F_t = ma$ ①

KOLO: $\sum \vec{M} = J\vec{\alpha}$

z: $\quad M_m - F_t R = J\alpha$ ② $\quad J = \frac{m_s R^2}{2}$ ↑ valj.

KOLO NE SPODRSAVA: $\quad v = -\omega R \;/\frac{d}{dt}$

$a = -\alpha R$ ③

2 MOŽNOSTI

SPLOŠNO:

② $\Rightarrow$ $\quad F_t = \frac{1}{R}(M_m - J\alpha) = \frac{M_m}{R} + \frac{1}{R}\frac{m_s R^2}{2}\frac{a}{R}$ $\quad$ (J ↓ ③)

$F_t = \frac{M_m}{R} + \frac{m_s a}{2}$

① $\longrightarrow$ $\quad mg\sin\varphi - N_s\left[\frac{M_m}{R} + \frac{m_s a}{2}\right] = ma$

$a\left(1 + \frac{m_s}{m}\frac{N_s}{2}\right) = g\sin\varphi - \frac{N_s\alpha}{mR}v^2$

$\frac{m_s}{m}\frac{N_s}{2} \approx 0$ $\quad$ ($m_s \ll m$)

$J \approx 0$:

② $\Rightarrow$ $\quad F_t = \frac{M_m}{R}$

① $\longrightarrow$ $\quad mg\sin\varphi - N_s\frac{M_m}{R} = ma$

$a = g\sin\varphi - \frac{N_s\alpha}{mR}v^2$

$\Rightarrow$ $\quad a = g\sin\varphi - \frac{N_s\alpha}{mR}v^2$ (5)

$A = g\sin\varphi$

$B = \frac{N_s\alpha}{mR}$

$a = A - Bv^2$

$\frac{dv}{dt} = A - Bv^2$

$\frac{1}{A - Bv^2}dv = dt\frac{dx}{dx} = \frac{dx}{v}$ (5)

$\int_0^v \frac{v}{A - Bv^2}dv = \int_0^x dx$ (5)

$-\frac{1}{2B}\ln\frac{A - Bv^2}{A} = x$ (5)

$1 - \frac{B}{A}v^2 = e^{-2Bx}$

$v = \sqrt{\frac{A}{B}\left(1 - e^{-2Bx}\right)} = \sqrt{\frac{g\sin\varphi\, mR}{N_s\alpha}\left(1 - e^{-\frac{2N_s\alpha}{mR}x}\right)}$ (5)