Euler $y_{n+1} = y_n + h\, y' = y_n + h \cdot f(x_n, y_n)$

$y(0.1) = y(0) + 0.1 \cdot f(0,1)$

$= 1 + 0.1 \cdot (0^2 + 1^2) = \underline{1 + 0.1} = \underline{\underline{1.1}}$

$y(0.2) = y(0.1) + 0.1\, f(0.1, 1.1)$

$= 1.1 + 0.1\,(0.1^2 + 1.1^2) = \dots$

b) c:

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & \\ 1 & -1 & 2 & 0 \\ \hline & 1/6 & 2/3 & 1/6 \\ & k_0 & k_1 & k_2 \end{array}$$

(x_n, y_n)

$(x_n + \frac{1}{2} \cdot h,\ y_n + \frac{1}{2} \cdot k_1)$

$k_1 = h \cdot f(x_n, y_n)$

$= 0.1\, \underbrace{f(0, 1)}_{0^2+1^2} = \underline{\underline{0.1}}$

$(x_n + h,\ y_n - k_1 + 2k_2)$

$y(0.1) \approx y(0) + \frac{1}{2} \cdot 0.1$

$= 1 + 0.05$

$= 1.05$

$k_2 = h \cdot f(\frac{0.1}{2}, 1.05)$

$= 0.1\,(0.1^2 + 1.05^2)$

$= \underline{\underline{0.11125}}$

$y(0.1) = y(0) + \frac{1}{6} k_1 + \frac{2}{3} k_2 + \frac{1}{6} k_3$

$= 1 + \frac{1}{6} \cdot 0.1 + \frac{2}{3} \cdot 0.1125$

$k_3 = h \cdot f(0.1$

$k_1 = h \cdot f(x_n, y_n) = 0.1 \cdot \overbrace{f(0, 1)}^{0^2 + 1^2 = 1^2} = \underline{\underline{0.1}}$

$k_2 = h \cdot f(x_n + \frac{h}{2}, y_n + \frac{1}{2} k_1) = 0.1\, f(0.05, \underbrace{1 + 0.05}) = \underbrace{0.05^2 + 1.05^2} = 0.1105$

$k_3 = h \cdot f(x_n + h, y_n - k_1 + 2k_2)$

$= 0.1\, f(0.1, 1 - 0.1 + 2 \cdot 0.1105)$

$\underbrace{0.1^2 + (1 - 0.1 + 2 \cdot 0.1105)^2}_{0.12664}$

$y_{n+1} = y(0.1) = y(0) + \frac{1}{6} k_1 + \frac{2}{3} k_2 + \frac{1}{6} k_3$

$= 1 + \frac{1}{6} \cdot 0.1 + \frac{2}{3} \cdot 0.1105 + \frac{1}{6} \cdot 0.12664$

$= \underline{\underline{1.11144}}$