ODVODI

$f(x) = \ln(\ln(x)) + (\arcsin(3x))^2$

$f'(x) = ?$

$(f \circ g)'(x) = f'(g(x))\, g'(x)$ $\qquad \arcsin' x = \frac{1}{\sqrt{1-x^2}}$

$f'(x) = \frac{1}{\ln x} \frac{1}{x} + 2 \arcsin 3x \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 =$

$= \frac{1}{\ln x} + \frac{6 \arcsin 3x}{\sqrt{1-(3x)^2}}$

$x^2 + \frac{y^2}{4} = 1 \quad / d$

$2x\,dx + \frac{\cancel{2}y\,dy}{\cancel{4}2} = 1$ $\qquad y^2 = 4 - 4x^2$

$2x + \frac{1}{2} y y' = 1$ $\qquad y = 2\sqrt{1-x^2}$

$\frac{1}{2} y y' = 1 - 2x$

$y' = \frac{2(1-2x)}{y} = \frac{\cancel{2}(1-2x)}{\cancel{2}\sqrt{1-x^2}} = \frac{(1-2x)}{\sqrt{1-x^2}}$

INTEGRALI

1.) $\int\left(\frac{e^x}{e^{x+2}}-3^x\right)dx=\int\left(\frac{1}{e^2}-3^x\right)dx=\frac{x}{e^2}-\frac{3^x}{\ln 3}+C$

$$\int a^x=\frac{a^x}{\ln a}$$

2.) $\int\frac{\sin 2x}{\sin x}dx=\int\frac{2\sin x\cos x}{\sin x}dx=2\sin x$

3.) $\int\cos x\sqrt{\sin x}\,dx=\int\sqrt{t}=\frac{2}{3}t^{\frac{3}{2}}+C=\frac{2}{3}(\sin x)^{\frac{3}{2}}+C$

$t=\sin x$

$dt=\cos x\,dx$

4.) $\int\left(\frac{x^2+\ln x}{x}\right)dx=\frac{x^2}{2}+\int\frac{\ln x}{x}dx=\frac{x^2}{2}+\frac{\ln^2 x}{2}+C$

$t=\ln x$

$dt=\frac{1}{x}dx$

5.) $\int x^2\sin 3x\,dx=-\frac{1}{3}x^2\cos 3x+\int\frac{2x\cos 3x}{3}dx$

$u=x^2 \quad dv=\sin 3x\,dx$

$du=2x\,dx \quad v=-\frac{1}{3}\cos 3x$

$$= -x^2 \cdot \frac{\cos 3x}{3} + \frac{2x \sin 3x}{9} - \frac{2}{9}\int \sin 3x \, dx =$$

$$= -x^2 \cdot \frac{\cos 3x}{3} + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x =$$

$$= \frac{2}{9} x \sin 3x - \frac{\cos 3x}{27}\left(9x^2 - 2\right)$$

6.) $$\int \frac{\sqrt{x-1}}{3x} dx = \frac{1}{3}\int \frac{\sqrt{u}}{u+1} du = \int \frac{2t^2 dt}{3(t^2+1)} =$$

$\sqrt{u} = t$

$\frac{1}{\sqrt{u}} \cdot \frac{1}{2} du = dt$

$du = 2\sqrt{u} dt$

$du = 2t dt$

$$= \frac{2}{3} \int \frac{t^2 dt}{t^2+1} =$$

$$= \frac{2}{3} \int \left(1 + \frac{1}{t^2+1}\right) dt =$$

$(t^2):(t^2+1) = 1 - \frac{1}{t^2+1}$

$-t^2 - 1$

$\frac{-1}{t^2+1}$

$$= \frac{2}{3}\left(t - \arctan t\right) + C =$$

$$= \frac{2}{3}\left(\sqrt{x-1} - \arctan \sqrt{x-1} + C\right.$$

$t = \sqrt{u} = \sqrt{x-1}$

7.)

$$\int \frac{4x}{(x^2+3)(x+1)}\, dx = (1)$$

$$\frac{4x}{(x^2+3)(x+1)} = \frac{A+Bx}{x^2+3} + \frac{C}{x+1} = \frac{Ax+A+Bx^2+Bx+Cx^2+3C}{(x^2+3)(x+1)}$$

$$4x = (3C+A) + (A+B)x + (B+C)x^2$$

$3C+A=0$ $\quad A+B=4 \quad B=-C$

$A=-3C \quad -3C-C=4 \quad \underline{B=1}$

$\underline{A=3} \quad \underline{C=-1}$

$$(1) = \int \left(\frac{x+3}{x^2+3} - \frac{1}{x+1} \right) dx =$$

$$= \int \left(\underbrace{-\frac{1}{x+1}}_{(2)} + \underbrace{\frac{3}{x^2+3}}_{(3)} + \underbrace{\frac{x}{x^2+3}}_{(4)} \right) dx$$

(2) $$\int \left(-\frac{1}{x+1}\right) dx = -\ln|x+1| + C$$

(3) $$\int \frac{3}{x^2+3}\, dx = \frac{3}{3}\int \frac{1}{1+\frac{x^2}{3}}\, dx = \sqrt{3}\int \frac{1}{1+t^2}$$

$$t^2 = \frac{x^2}{3} \qquad = \sqrt{3}\, \operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) + D$$

$$t = \frac{x}{\sqrt{3}}$$

$$dt = \frac{1}{\sqrt{3}}\, dx$$

$$(4)\quad \int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2+3| + E$$

$$u = x^2+3$$

$$du = 2x\,dx$$

$$\frac{1}{2}\,du = x\,dx$$

REŠITEV:

$$\int \frac{4x}{(x^2+3)(x+1)} = (2)+(3)+(4) = \qquad \tilde{C} := C+1)+E$$

$$= -\ln|x+1| + \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2}\ln|x^2+3| + \underbrace{C+1)+E}_{\tilde{C}}$$

določimo iz začetnega pogoja