Topological Data Analysis
Lab work, 4th week

1. For each of the following triangulations determine if it is a triangulation of a surface (a 2-dimensional manifold).

A: `[(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)]`

B: `[(1, 2, 3), (1, 2, 4), (2, 3, 5), (2, 3, 6), (3, 5, 7)]`

C:
```
[(1, 2, 3), (2, 3, 4), (3, 4, 5),
 (4, 5, 6), (1, 5, 6), (1, 2, 6)]
```

D:
```
[(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),
 (1, 4, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),
 (4, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),
 (2, 3, 9), (3, 5, 9), (1, 3, 5)]
```

E:
```
[(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),
 (1, 5, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),
 (5, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),
 (2, 3, 9), (3, 4, 9), (1, 3, 4)]
```

F: `[(1, 2, 3), (1, 3, 4), (2, 3, 4), (4, 5, 6)]`

G: `[(1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (2, 5, 6), (1, 2, 6)]`

H:
```
[(1, 3, 5), (1, 2, 6), (1, 5, 6), (1, 2, 4), (1, 3, 4),
 (2, 3, 5), (2, 3, 6), (2, 4, 5), (3, 4, 6), (4, 5, 6)]
```

   (a) Find the Euler characteristics for all of these simplicial complexes.

   (b) For each simplicial complex find the link of each vertex. Is this simplicial complex a manifold?

   (c) Find the number of boundary components for all of the manifolds.

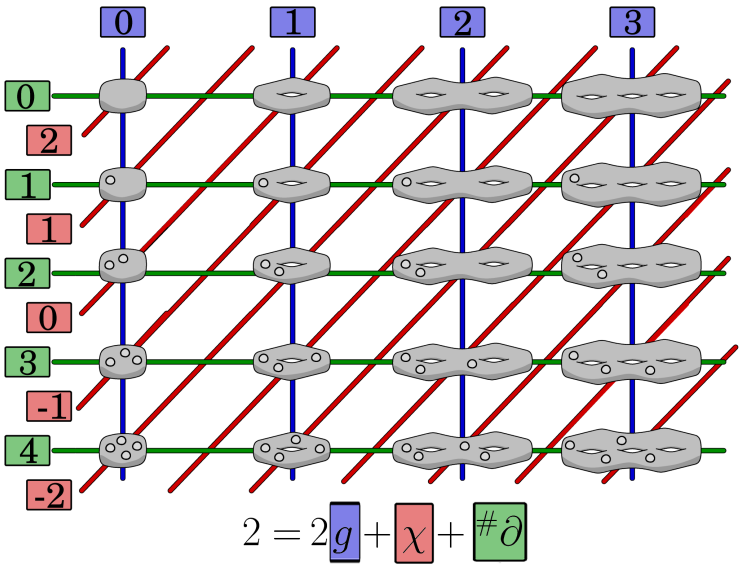
   (d) For each manifold determine if it is orientable or not.

   (e) Determine the genus of each orientable surface and the genus of non-orientable surfaces with no boundary.

   (f) Identify each of the surfaces.

Use the following array to keep track of the results.

	Euler characteristic	manifold Y/N	# of boundary components	orientable Y/N	genus	name
A						
B						
C						
D						
E						
F						
G						
H						



genus	0	1	2
orientable	S^2	T	$T\#T$
non-orientable		P	$P\#P$

2. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Vietoris-Rips complex $\mathrm{VR}(S,R)$ for

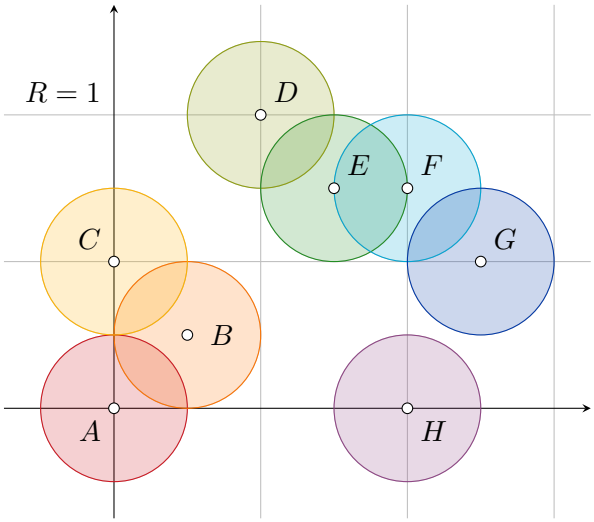
   (a) $R = 1$,

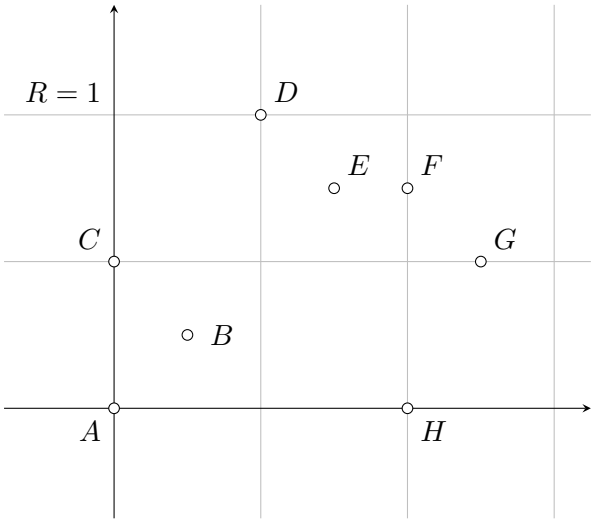
   (b) $R = 1.2$,

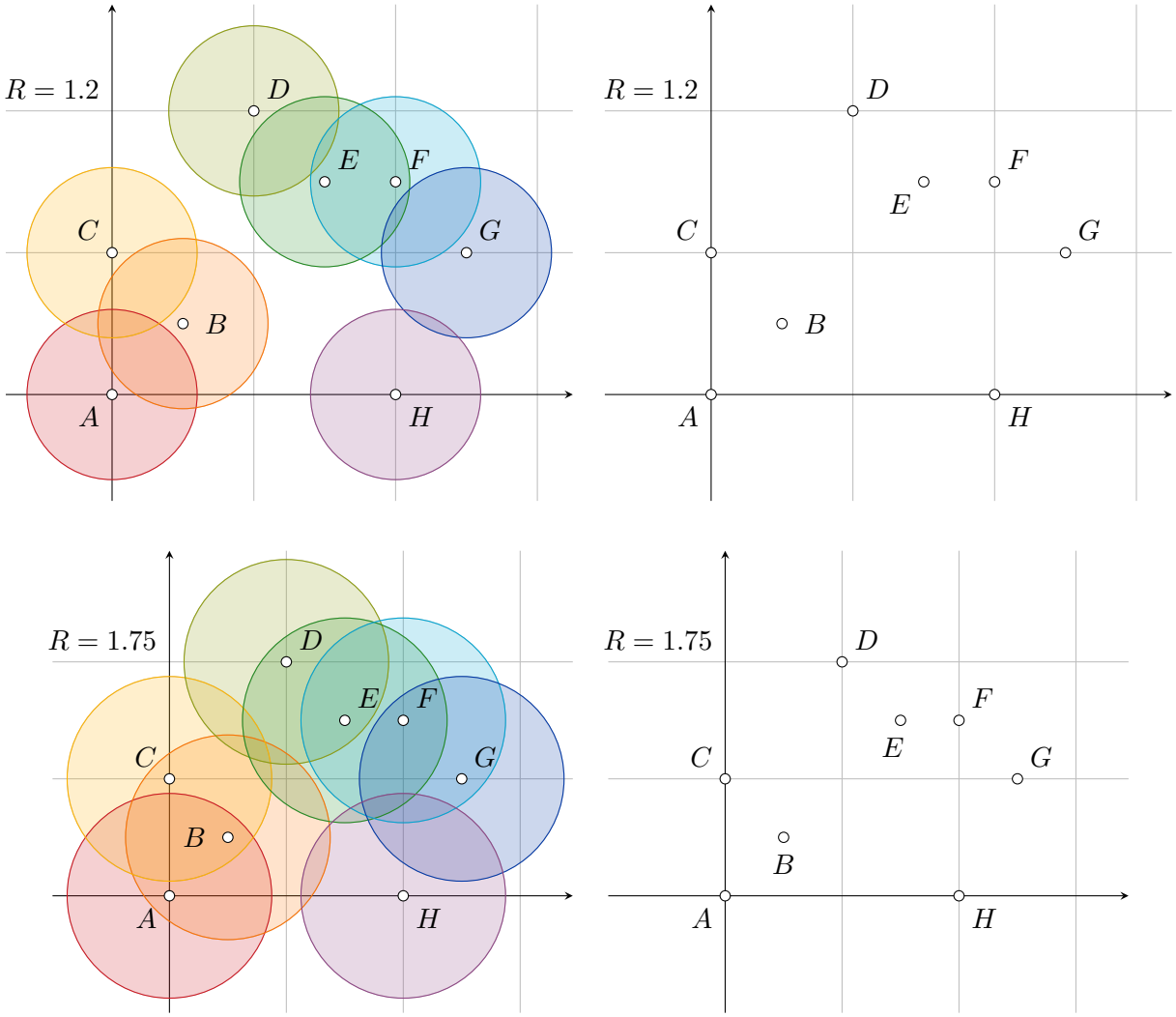
   (c) $R = 1.75$.

   In each case list all the simplices and determine its dimension.

   Assuming there is a sensor placed at each point of S and all sensors can detect points that are at distance 1.75 or less, is the area covered by the sensors connected? Does it contain any holes?







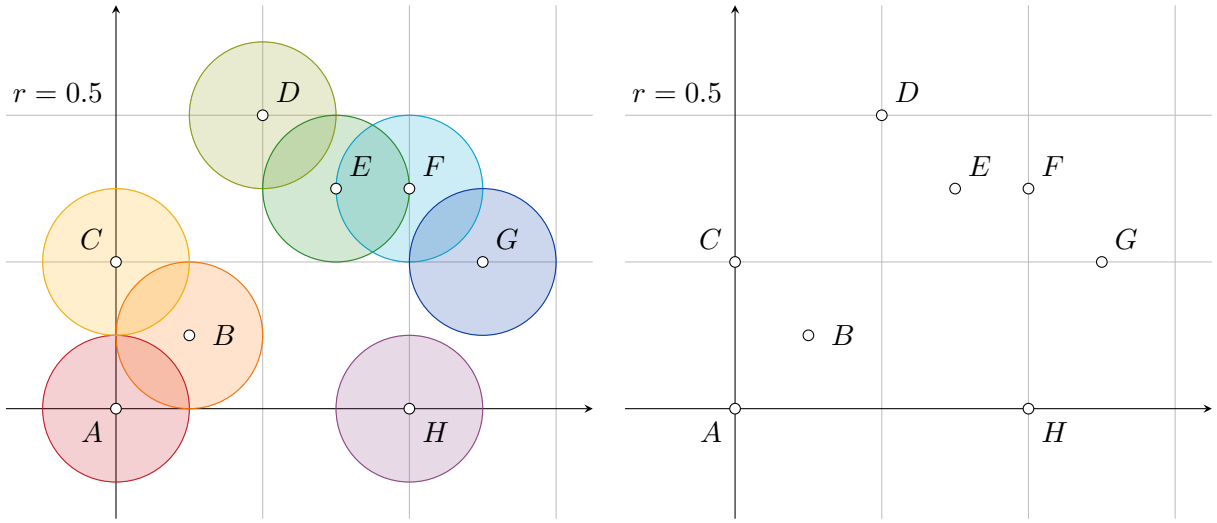
3. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Čech complex $\mathrm{C}(S,r)$ for

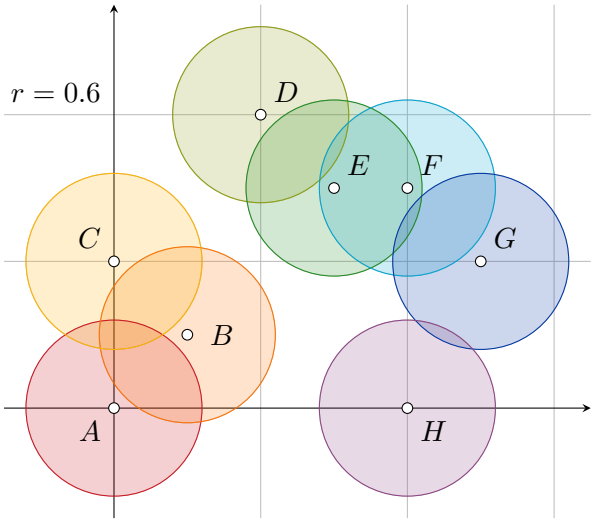
   (a) $r = 0.5$,

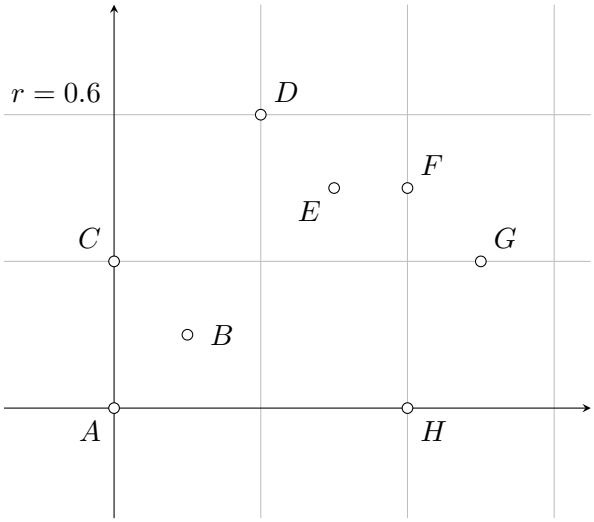
   (b) $r = 0.6$,

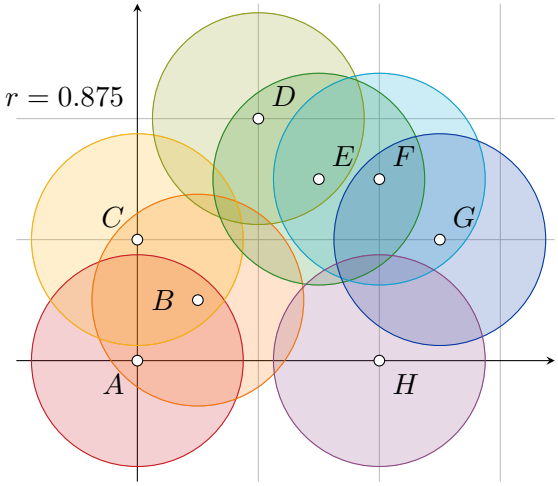
   (c) $r = 0.875$.

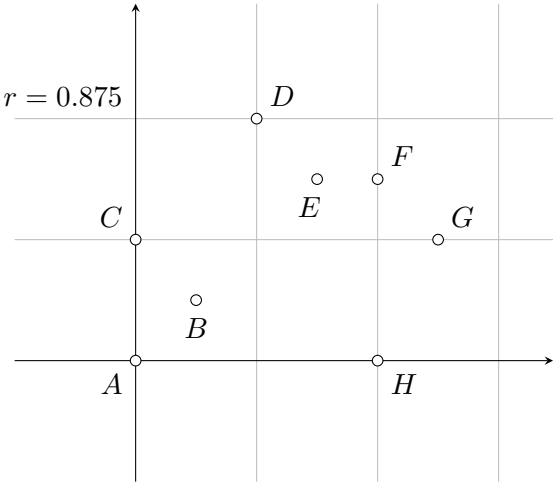
   In each case list all the simplices and determine its dimension.




$r = 0.6$
D
E
F
C
G
B
A
H


$r = 0.6$
D
F
E
C
G
B
A
H


$r = 0.875$
D
E
F
C
G
B
A
H


$r = 0.875$
D
F
E
C
G
B
A
H