$\forall x P(x) \land \forall x Q(x) \sim$

$\forall x \, (P(x) \land Q(x))$

$\forall x P(x) \land \forall y Q(y) \sim$

$\forall x \, (P(x) \land \forall y Q(y)) \sim$

$\forall x \forall y \, (P(x) \land Q(y))$

Levo in desno stran I. ... $\cap A^c$

$$\text{I} \qquad A \cup X = B \qquad\qquad (A \cup X) \cap A^c = B \cap A^c$$

$$\text{II} \qquad \underbrace{A \cap X}_{} = C \qquad\qquad \cancel{(A \cap A^c)}^{\emptyset} \cup (X \cap A^c) = X \cap A^c$$

iz I sledi $\quad A \subseteq B \quad$ in $\quad X \subseteq B$

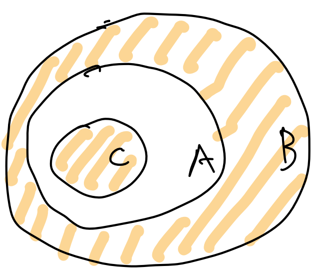
iz II sledi $\quad C \subseteq A \quad$ in $\quad C \subseteq X$

$$C \subseteq A \subseteq B$$

univerzalna množica

$$X = X \cap S = X \cap (A \cup A^c) = (X \cap A) \cup (X \cap A^c) =$$

$$= C \cup B \cap A^c$$



Preizkus.

$$\text{I.} \quad A \cup X = A \cup C \cup B \cap A^c =$$
$$= A \cup B \cap A^c =$$
$$= A \cup B \cap A \cup A^c =$$
$$= A \cup B = B \quad ✓$$

$$\text{II.} \quad A \cap X = A \cap (C \cup B \cap A^c) =$$
$$= A \cap C \cup \cancel{A \cap B \cap A^c}^{\emptyset}$$
$$= A \cap C = C \quad ✓$$

Dokaz. $(\Leftarrow)$ privzamemo, da $A=\emptyset$ in $B=\emptyset$

$$A\cup B = \emptyset \cup \emptyset = \emptyset$$

$(\Rightarrow)$ privzamemo, da $A\cup B=\emptyset$

$$A \subseteq A\cup B = \emptyset \implies A=\emptyset$$
$$B \subseteq A\cup B = \emptyset \implies B=\emptyset$$

$\blacksquare$

Dokaz. $(\Leftarrow)$ privzamemo, da $A=B$.

$$A+B = (A\cup B)\setminus(A\cap B) = (A\cup A)\setminus(A\cap A) = A\setminus A = \emptyset$$

$(\Rightarrow)$ privzamemo, da $A+B=\emptyset$

$$\emptyset = A+B = A\setminus B \cup B\setminus A$$

Po zg. trditvi velja $A\setminus B=\emptyset$ in $B\setminus A=\emptyset$.

$$\left.\begin{array}{l} A\setminus B=\emptyset \implies A\subseteq B \\ B\setminus A=\emptyset \implies B\subseteq A \end{array}\right\} A=B$$

$\blacksquare$

Trditev $\quad A\subseteq B \iff A\cup B = B$

Dokaz (vemo iz enakosti z množicami)

$$\begin{array}{ll} \text{I.} & A\cap X = B\setminus X \\ \text{II} & C\cup X = X\setminus A \end{array}$$

$$\begin{array}{ll} \text{I} & (A\cap X)+(B\setminus X) = \emptyset \\ \text{II} & (C\cup X)+(X\setminus A) = \emptyset \end{array}$$

$$\emptyset = ((A\cap X)+(B\setminus X)) \cup ((C\cup X)+(X\setminus A))$$

$$= (A \cap X \cap (B \cap X^c)^c) \cup (B \cap X^c \cap (A \cap X)^c) \cup$$

$$((C \cup X) \cap (X \cap A^c)^c) \cup (X \cap A^c \cap (C \cup X)^c) =$$

$$= (A \cap \underbrace{X \cap (B^c \cup X)}_{abs.}^{X}) \cup (B \cap \underbrace{X^c \cap (A^c \cup X^c)}_{abs}^{X^c}) \cup$$

$$((C \cup X) \cap (X^c \cup A)) \cup \underbrace{\cancel{(X \cap A^c \cap C^c \cap X^c)}}_{\emptyset} =$$

$$= (A \cap X) \cup (B \cap X^c) \cup (C \cap X^c) \cup (C \cap A) \cup$$

$$\underbrace{\cancel{(X \cap X^c)}}_{\emptyset} \cup \underbrace{\cancel{(X \cap A)}}_{idem} =$$

$$= (X \cap A) \cup (X^c \cap B) \cup (X^c \cap C) \cup (C \cap A) =$$

$$C \cap A =$$

$$C \cap A \cap S = C \cap A \cap (X \cup X^c) =$$

$$(C \cap A \cap X) \cup (C \cap A \cap X^c)$$

$$= (X \cap A) \cup (X^c \cap B) \cup (X^c \cap C) \cup \underbrace{\cancel{(X \cap A \cap C)}}_{abs} \cup \underbrace{\cancel{(X^c \cap A \cap C)}}_{abs} =$$

$$= (X \cap A) \cup (X^c \cap B) \cup (X^c \cap C)$$

$$= (X \cap A) \cup (X^c \cap (B \cup C))$$

$$= (X \cap P) \cup (X^c \cap Q) = \emptyset$$

P, Q odvisni od A, B, C

$$\downarrow \qquad\qquad \downarrow$$

$$X \subseteq P^c \qquad\qquad Q \subseteq X$$

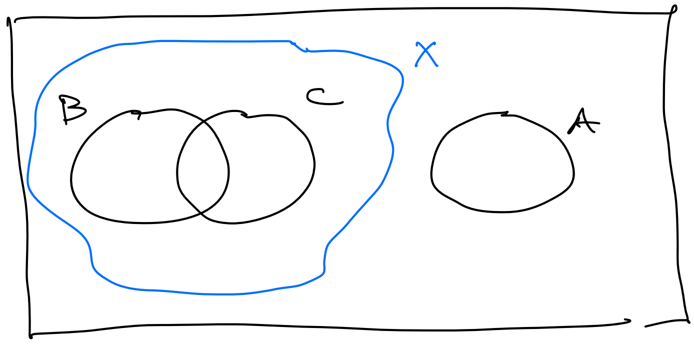
$$Q \subseteq X \subseteq P^c \qquad\qquad Q \subseteq P^c$$

$\emptyset = (X \cap A) \cup (X^c \cap (B \cup C))$ $\qquad (B \cup C) \cap A = \emptyset$

Pogoj za rešljivost $\qquad B \cup C \subseteq A^c$

Rešitve

$B \cup C \subseteq X \subseteq A^c$



Parametrično

$X = (B \cup C) \cup (T \cap A^c)$

$$\left.\begin{array}{l} \emptyset \subseteq A \\ A \subseteq A \end{array}\right\} \Longrightarrow \begin{array}{l} \emptyset \in \mathcal{P}A \\ A \in \mathcal{P}A \end{array}$$

$\mathcal{P}\emptyset \qquad \emptyset \in \mathcal{P}\emptyset \qquad \mathcal{P}\emptyset = \{\emptyset\}$

$\mathcal{P}\{\emptyset\} \qquad \mathcal{P}\{\emptyset\} = \{\emptyset, \{\emptyset\}\}$

$\mathcal{P}\{1,2,3\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

podmnožice $\{1,2,3\}$ bi lahko opisali z logičnimi vrednostmi treh izjavnih spremenljivk p_1, p_2, p_3

p_i .... ali i vtaknjen v podmnožico ali ne.

$\mathcal{A} = \{A_1, A_2, A_3\}$

$A_1 = \{a, b\}$
$A_2 = \{b, c\}$
$A_3 = \{b, d, e\}$

$\mathcal{A} = \{ \{a,b\}, \{b,c\}, \{b,d,e\} \}$

$$\bigcup \mathcal{A} = \bigcup_{i \in I} A_i = \bigcup_{i=1}^{3} A_i = \{a, b, b, c, b, d, e\} =$$
$$= \{a, b, c, d, e\}$$

$$\bigcap \mathcal{A} = \bigcap_{i \in I} A_i = \bigcap_{i=1}^{3} A_i = \{b\}$$

---

intervali na $\mathbb{R}$

$$[a,b] = \{x \in \mathbb{R},\ a \leq x \leq b\}$$
$$(a,b) = \{x \in \mathbb{R},\ a < x < b\}$$
$$(a,b] = \{x \in \mathbb{R},\ a < x \leq b\}$$
$$(a,\infty) = \{x \in \mathbb{R},\ a < x\}$$
$$(-\infty,b] = \{x \in \mathbb{R},\ x \leq b\}$$

- - - - - - - - - - - -

$$\bigcup_{i \in I} A_i = \bigcup_{i \in \mathbb{Z}} [i, i+1] = \mathbb{R}$$

$$\bigcap_{i \in I} A_i = \bigcap_{i \in \mathbb{Z}} [i, i+1] = \emptyset$$

- - - - - - - -

$$B_0 \supseteq B_1 \supseteq B_2 \supseteq B_3 \supseteq \dots$$

$$\bigcup_{i\in\mathbb{N}} B_i = \bigcup_{i\in\mathbb{N}} (i,\infty) =$$

$$= \bigcup_{i=0}^{\infty} (i,\infty) = (0,\infty) = B_0$$

$$\bigcap_{i\in\mathbb{N}} (i,\infty) = \emptyset$$

$$C_0 \subseteq C_1 \subseteq C_2 \subseteq C_3 \subseteq \dots,$$

$$\bigcup_{i\in\mathbb{N}} C_i = \bigcup_{i=0}^{\infty} (2^{-i},\infty) = (0,\infty)$$

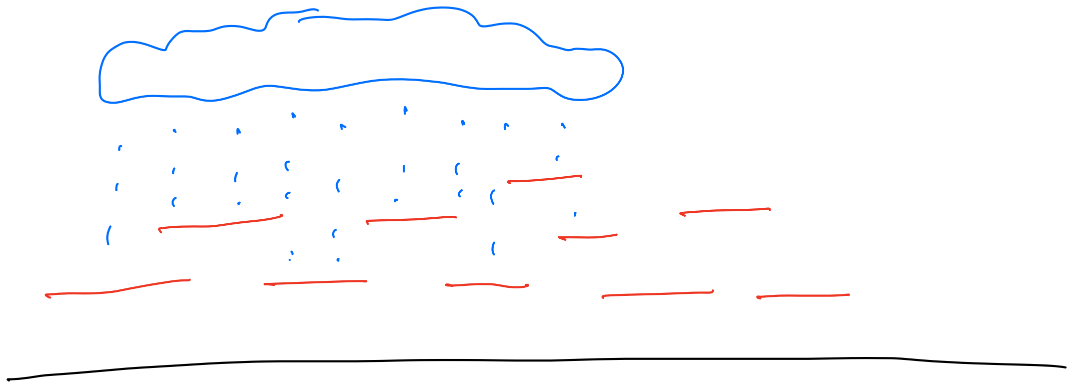
$$\bigcap_{i\in\mathbb{N}} C_i = \bigcap_{i=0}^{\infty} (2^{-i},\infty) = (1,\infty) = C_0$$

$$\bigcup_{i=1}^{\infty} D_i = \bigcup_{i=1}^{\infty} (1/i, 2+\tfrac{1}{i}] = (0,3]$$

$$\bigcap_{i=1}^{\infty} D_i = \bigcap_{i=1}^{\infty} (1/i, 2+\tfrac{1}{i}] = (1,2]$$

$$\bigcup_{i\in\mathbb{Z}} A'_i = \bigcup_{i\in\mathbb{Z}} [i,i+1) = \mathbb{R}$$

$$\bigcap_{i\in\mathbb{Z}} A'_i = \emptyset$$



Komentar.

Včasih (v kakšni drugi literaturi) pravijo, da je

$\mathcal{A} = \{A_i;\ i \in I\}$ pokritje B tudi, če

je $B \subseteq \bigcup_{i\in I} A_i$ (in ne nujno $B = \bigcup_{i\in I} A_i$)

$\{a,b\}$ .... par a,b

(a,b)

(a,a) je urejen par, v katerem sta obe koordinati enaki a.

Trditev pomeni isto kot

$\{\{a\},\{a,b\}\} = \{\{c\},\{c,d\}\}$ natanko tedaj, ko $a=c$ in $b=d$

$(\Leftarrow)$ ✓

$(\Rightarrow)$ $\{a\} \in \{\{c\},\{c,d\}\}$

$\{a\} = \{c\}$ ali $\{a\} = \{c,d\}$ ....

.... doma naprej.

$A = \{1, 2, 3\}$

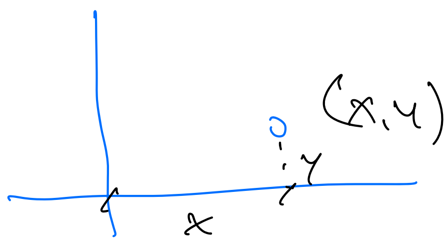
$B = \{a, b\}$

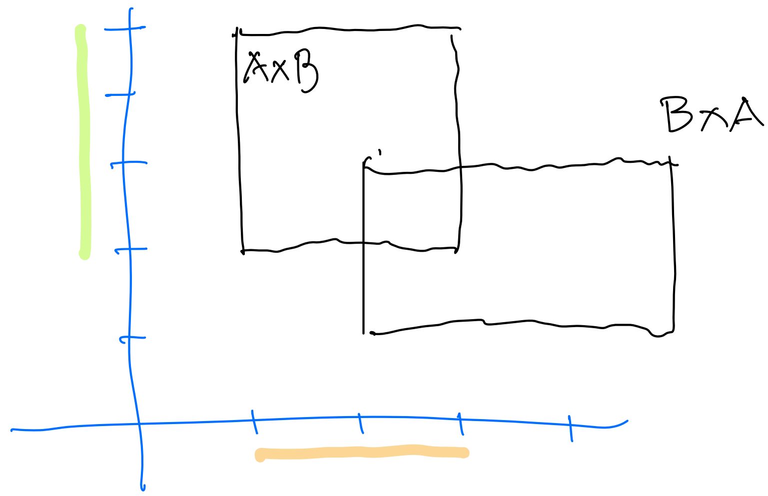
$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}.$

$$A \times B \neq B \times A$$

Descartes





$A = [1,3]$

$B = [2,5]$

$$(A \times B) \times C \cong A \times (B \times C) =: A \times B \times C$$

$A \times A \times A = A^3$ množica zaporedij dolžine 3 s členi iz A

Za $\times$ in $\cap, \cup$ nočemo definirati relativnih prednosti

$A \times (B \cup C)$ $\qquad$ $(A \times B) \cup C$ $\qquad$ $A \times B \cup C$

Ali 1 in 2 veljata tudi za $\cap$? DA.

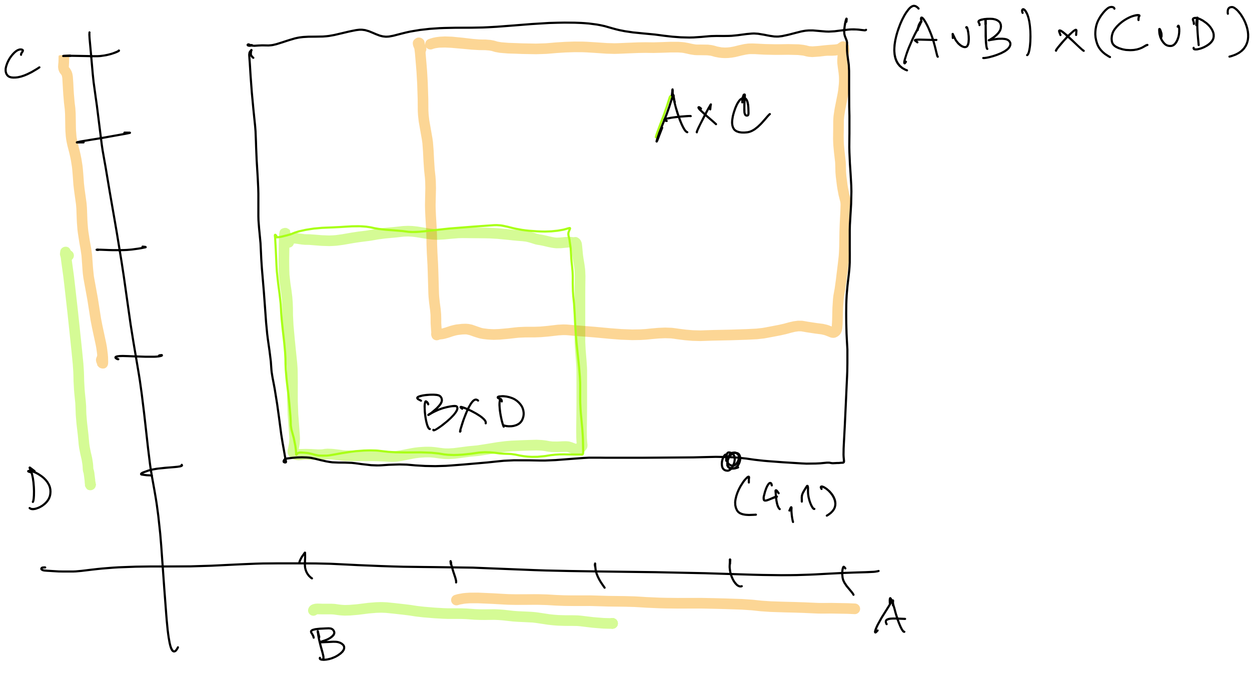
Ali 3 velja tudi za $\cup$? NE.

$$(A \cap B) \times C = (A \cap B) \times (C \cap C) =$$
$$= (A \times C) \cap (B \times C)$$

Poiščimo protiprimer, množica A, B, C, D, za katere velja

$$(A \cup B) \times (C \cup D) \neq (A \times C) \cup (B \times D)$$

$(4,1) \in$ $\qquad$ $(4,1) \notin$



$A = C = [2,5]$

$B = D = [1,3]$

Protiprimer za ($\Leftarrow$) v primeru, ko $A\times B = \emptyset$

$A\times B \subseteq C\times D$ vedno velja

$A = \emptyset$
$C = D = \{1,2\}$
$B = \mathbb{N}$

Res je sicer, da $A \subseteq C$, toda $B \not\subseteq D$.

Namesto

$(a,b)\in R$

uporabljamo relacijski zapis

vmesni

aRb

a je v relaciji R z b-jem

~~$(a,b)\in R$~~

aRb natanko tedaj, ko $a \leq b$

relacija $\leq$ v $\mathbb{N}$

$$\leq \; = \{(0,0),(0,1),(0,2),(0,3),\ldots,(1,1),(1,2),\ldots,(2,2),(2,3),\ldots,\ldots\}$$

je kot množica enaka

$$= \{(x,y),\; x,y\in\mathbb{N},\; \exists z\in\mathbb{N},\; y = x+z\}$$

namesto id_A bomo uporabljali $=$

enakost elementov v množici A

$= \; = \{(x,x),\; x \in A\}$

je tudi množica

1. $D_R = \{a, b, c\}$ $(\subset A)$

$Z_R = \{b, c, d, a\}$ $(= A)$