$\checkmark = 3{,}125$

1.

(a) $v(t) = v_0 + at$ ✓

$t_1 = 2{,}7s \rightarrow v(t_1) = 0\,m/s + 0{,}25\,m/s^2 \cdot 2{,}7s = 0{,}675\,m/s$ ✓

$t_2 = 7{,}3s \rightarrow v(4s) = 0{,}25\,m/s^2 \cdot 4s = 1\,m/s$

$v(7{,}3s) = 1\,m/s - 0{,}75\,m/s^2 \cdot (7{,}3s - 4s) = -1{,}475\,m/s$ ✓

(b) $x(t) = x_0 + v_0 t + \frac{at^2}{2}$ ✓ , $t = 14{,}5s$

i) $t = 0 \rightarrow 4s$: $x(4s) = \frac{1}{2}\, 0{,}25\,m/s^2 \cdot (4s)^2 = +2m$

$v(4s) = 1\,m/s$

ii) $t = 4s \rightarrow 8s$: $x(8s) = 2m + 1\,m/s \cdot 4s - \frac{1}{2}\, 0{,}75\,m/s^2 \cdot (4s)^2 = 0m$

$v(8s) = 1\,m/s - 0{,}75\,m/s^2 \cdot 4s = -2\,m/s$

iii) $t = 8s \rightarrow 14{,}5s$: $x(14{,}5s) = 0m - 2\,m/s \cdot 6{,}5s + \frac{1}{2}\, 0{,}5\,m/s^2 \cdot (6{,}5s)^2 =$

$14{,}5s - 8s = 6{,}5s$

$= -2{,}44\,m$ ✓ položaj.

Pot : telo spremeni smer gibanja (hitrost spremeni predznak) pri:

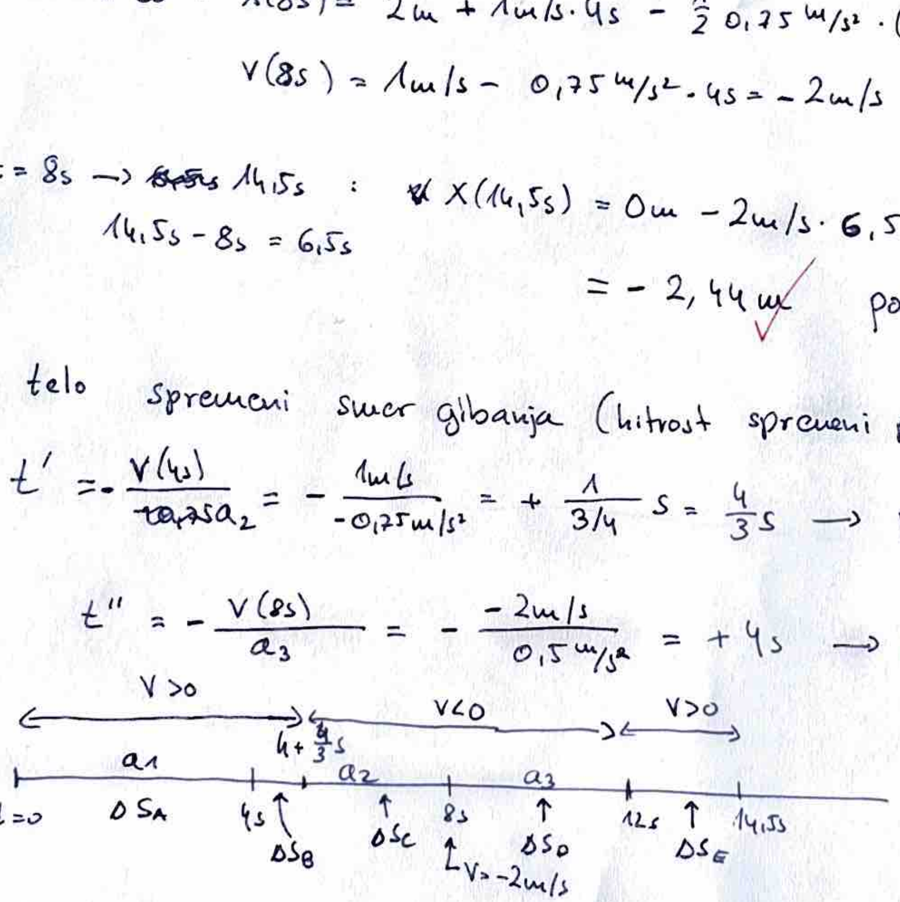
$v = at + v_0$

$v = 0$

$t = -\frac{v_0}{a}$

$t' = -\frac{v(4s)}{a_2} = -\frac{1\,m/s}{-0{,}75\,m/s^2} = +\frac{1}{3/4}\,s = \frac{4}{3}s \rightarrow$ pri $4s + \frac{4}{3}s$

$t'' = -\frac{v(8s)}{a_3} = -\frac{-2\,m/s}{0{,}5\,m/s^2} = +4s \rightarrow$ pri $8s + 4s = 12s$



$\Delta S_A = \frac{1}{2}\, 0{,}25\,m/s^2 \cdot (4s)^2 = 2m$

$\Delta S_B = 1\,m/s \cdot \frac{4}{3}s - \frac{1}{2}\, 0{,}75\,m/s^2 \cdot \left(\frac{4}{3}s\right)^2 = \frac{2}{3}m$

$\Delta S_C = \left|0\,m/s \cdot \frac{8}{3}s - \frac{1}{2}\, 0{,}75\,m/s \cdot \left(\frac{8}{3}s\right)^2\right| = \frac{8}{3}m$

$(4 - \frac{4}{3})s = \frac{8}{3}s$

$\Delta S_D = \left|-2\,m/s \cdot 4s + \frac{1}{2}\, 0{,}5\,m/s^2 \cdot (4s)^2\right| = 4m$

$12s - 8s$

$\Delta S_E = 0\,m/s \cdot 2{,}5s + \frac{1}{2}\, 0{,}5\,m/s^2 \cdot (2{,}5s)^2 = 1{,}56m$

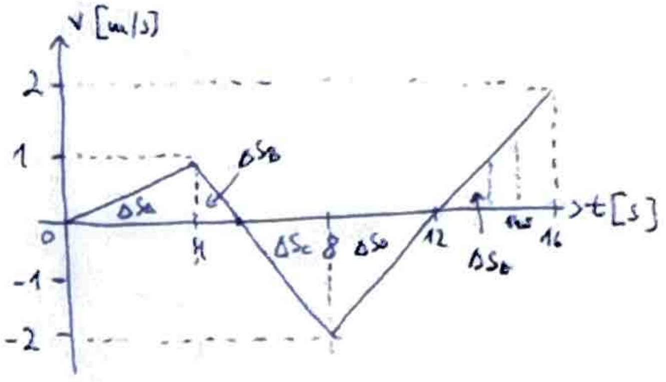
$14{,}5s - 12s = 2{,}5s$

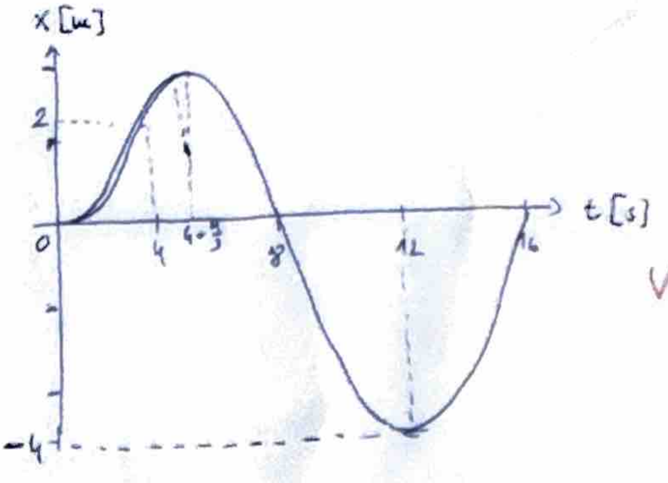
$s = \Delta S_A + \Delta S_B + \Delta S_C + \Delta S_D + \Delta S_E$

$\approx 10{,}9m$ ✓

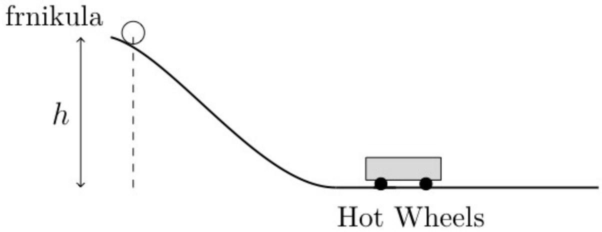
ALTERNATIVNO: Ploščina pod grafom $v(t)$

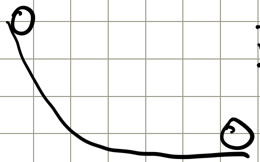
(c)





2. Frnikula z radijem 1 cm se brez spodrsavanja kotali po klancu. Ob vznožju klanca je parkiran Hot Wheels avtomobilček. Pri trku se avtomobilček in frnikula zlepita. Kako visoko (h) mora začeti frnikula, da je hitrost avtomobilčka po trku enaka 1,6 m/s? Masa frnikule znaša 20 g, masa avtomobilčka 30 g. Trenje zanemari.





ZOE: $\quad z: \; W_{kot}^{(z)} = mgh \quad$ 3

$K: \; W_{kot}^{(k)} = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2 \quad$ 3

kotaljenje: $\quad v = \omega r \quad$ 2

$J = \frac{2}{5} m r^2 \quad$ 2

$$W_{kot}^{(k)} = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{5} m v^2 = \frac{7}{10} m v^2$$

$$mgh = \frac{7}{10} m v^2 \quad \Rightarrow \quad v = \sqrt{\frac{10}{7} g h} \quad 2$$

neprožni trk: $\quad z: \; G_{tot}^{(z)} = m v + m_H \cdot 0 \quad$ 3

$K: \; G_{tot}^{(k)} = (m + m_H) v' \quad$ 3

ZOGK: $\quad m v = (m + m_H) v'$

$$v' = \frac{m}{m + m_H} v \quad 2$$

ZDRUŽIM:

$$v' = \frac{m}{m + m_H} \sqrt{\frac{10}{7} g h} \Bigg/^{2} \quad \text{in izrazim } h$$

$$v'^2 = \left(\frac{m}{m + m_H}\right)^2 \frac{10}{7} g h$$

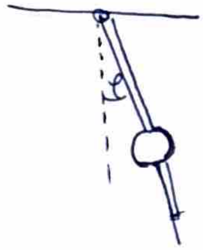
$$h = \left(\frac{m + m_H}{m}\right)^2 \cdot \frac{7}{10} \cdot \frac{1}{g} \cdot v'^2 \quad 3$$

$$h = \left(\frac{50\,g}{20\,g}\right)^2 \cdot \frac{7}{10} \cdot \frac{s^2}{10\,m} \cdot \left(1.6\,\frac{m}{s}\right) = \underline{\underline{1.12\,m}} \quad 2$$

⑪

$\sqrt{\ } = 3{,}125$

3.



$M_p = -m_p g \frac{l}{2} \sin\varphi$

$M_k = -m_k g l' \sin\varphi$

$J_p = \frac{1}{3} m_p l^2$

$J_k = \frac{2}{5} m_k r^2 + m_k l'^2$

$J\alpha = M$

$$\alpha = \frac{d^2\varphi}{dt^2} = -\frac{m_p g \frac{l}{2}\sin\varphi + m_k g l' \sin\varphi}{\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2}$$

$\sin\varphi \approx \varphi$

$$\frac{d^2\varphi}{dt^2} = -\underbrace{\frac{m_p g \frac{l}{2} + m_k g l'}{\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2}}_{=\omega^2}\varphi$$

$$\omega^2 = +\frac{m_p g l/2 + m_k g l'}{\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2} \quad (*)$$

$$\omega^2 \left(\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2\right) = m_p g \frac{l}{2} + m_k g l'$$

$$\frac{1}{3} m_p l^2 \omega^2 + \omega^2\left(\frac{2}{5} r^2 + l'^2\right) m_k = m_p g \frac{l}{2} + m_k g l'$$

$$m_k \cdot \omega^2 \left(\frac{2}{5} r^2 + l'^2\right) - m_k g l' = m_p g \frac{l}{2} - \frac{1}{3} m_p l^2 \omega^2$$

$$m_k \left[\omega^2\left(\frac{2}{5} r^2 + l'^2\right) - \cancel{m} g l'\right] = m_p g \frac{l}{2} - \frac{1}{3} m_p l^2 \omega^2$$

$$m_k = \frac{m_p g\, l/2 - \frac{1}{3} m_p l^2 \omega^2}{\omega^2\left(\frac{2}{5} r^2 + l'^2\right) - g l'}, \quad \omega = \frac{2\pi}{t_0}$$

$$= \frac{0{,}1\,\mathrm{kg} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2}\, \frac{1{,}05\,\mathrm{m}}{2} - \frac{1}{3}\, 0{,}1\,\mathrm{kg} \cdot (1{,}05\,\mathrm{m})^2 \cdot \left(\frac{2\pi}{1{,}6\,\mathrm{s}}\right)^2}{\left(\frac{2\pi}{1{,}6\,\mathrm{s}}\right)^2 \left[\frac{2}{5}(0{,}05\,\mathrm{m})^2 + (0{,}6\,\mathrm{m})^2\right] - 10\,\mathrm{m/s^2} \cdot (0{,}6\,\mathrm{m})} =$$

$$= \begin{cases} 0{,}1\,\mathrm{kg} & \text{z } g = 9{,}8\,\mathrm{m/s^2} \\ 0{,}09\,\mathrm{kg} & \text{z } g = 10\,\mathrm{m/s^2} \end{cases}$$

ALTERNATIVNO: ~~4π~~

$$\omega = \sqrt{\frac{m g r^*}{J}} \Rightarrow \omega^2 = \frac{\cancel{g(m_p + m_k)}\, \frac{m_p \frac{l}{2} + m_k l'}{\cancel{m_p + m_k}}}{\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2} =$$

$m = m_p + m_k$

$$\omega^2 = \frac{(m_p\, l/2 + m_k l')g}{\frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2} \quad (*)$$ enako naprej

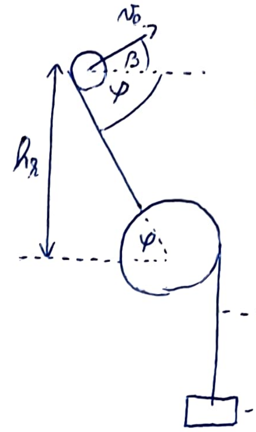
$$r^* = \frac{m_p\, l/2 + m_k l'}{m_p + m_k}$$

↑ položaj težišča

$$J = \frac{1}{3} m_p l^2 + \frac{2}{5} m_k r^2 + m_k l'^2$$

1. KOLOKVIJ 2025/26 - 4. naloga

NI DELA ZUNANJIH SIL $\Rightarrow$ ENERGIJA SE OHRANJA



$$\beta = \frac{\pi}{2} - \varphi$$

$$\Delta W_{p,u} + \Delta W_{k,u} + \Delta W_{p,g} + \Delta W_{rot} = 0 \quad 5$$

$$-m_u g h_u + \frac{m_u v_u^2}{2} + m_g g h_g + \frac{J\omega^2}{2} = 0 \quad (1)$$

$$h_u = R\varphi \;/\frac{d}{dt} \qquad h_g = (R+l)\sin\varphi$$

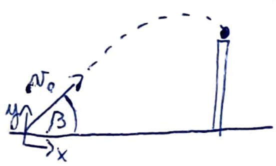
$$v_u = R\omega \quad 3 \qquad J = \frac{1}{2} m_v R^2 + m_g (R+l)^2 \quad 3$$

ZANIMA NAS HITROST $v_0 = (R+l)\omega$

$$(1):\quad -m_u g R\varphi + \frac{m_u R^2\omega^2}{2} + m_g g (R+l)\sin\varphi + \frac{J\omega^2}{2} = 0$$

$$\omega^2\left[\frac{1}{2} m_u R^2 + \frac{1}{2} J\right] = m_u g R\varphi - m_g g (R+l)\sin\varphi$$

$$\omega^2 = \frac{2g\left(m_u R\varphi - m_g (R+l)\sin\varphi\right)}{m_u R^2 + \frac{1}{2} m_v R^2 + m_g (R+l)^2} \quad 3$$



$$\Rightarrow \quad v_0 = (R+l)\sqrt{\frac{2g\left[m_u R\varphi - m_g (R+l)\sin\varphi\right]}{m_u R^2 + \frac{1}{2} m_v R^2 + m_g (R+l)^2}}$$

$$v_0 = 11{,}18\,\frac{m}{s} \quad 1$$

x - smer:

$$x = v_0 \cos\beta\, t \Rightarrow t = \frac{x}{v_0\cos\beta} \quad 3$$

y - smer:

$$y(t) = v_0 \sin\beta\, t - \frac{g t^2}{2} \quad 3 \xrightarrow{\text{VSTAVIMO}} y(x) = \cancel{v_0} \sin\beta \frac{x}{\cancel{v_0}\cos\beta} - \frac{g}{2}\frac{x^2}{v_0^2\cos^2\beta}$$

$$y(x) = x\tan\beta - \frac{g}{2v_0^2\cos^2\beta} x^2 \quad 3$$

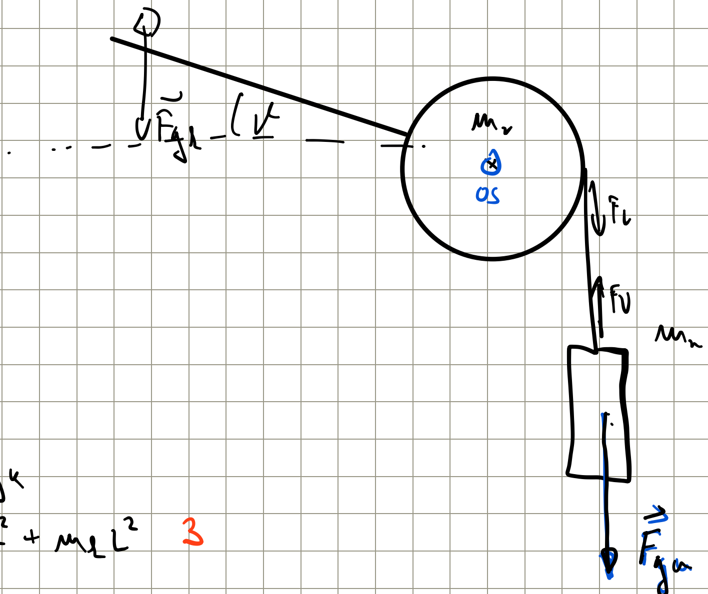
$$y(d) = d\tan\left(\frac{\pi}{2}-\varphi\right) - \frac{g}{2v_0^2\cos^2\left(\frac{\pi}{2}-\varphi\right)} d^2$$

$$y(d) = -2{,}98 \text{ m}$$

$\Downarrow$ POD VIŠINO OBZIDJA

$\Rightarrow$ KAMEN NE PRIDE ČEZ OBZIDJE 1 alt. reševanje z navori

2 NZN: $\sum M = J\alpha$



$J = J_r + J_k$

$= \frac{1}{2} m_r R^2 + m_2 L^2$ 3

enačba za utež: $(F = ma)$

$m_u g - F_v = m_u R \ddot{\vartheta}$ 3

še zrešimo F_v

enačba za vrteče telo $(M = J\alpha)$

$F_v \cdot R - m_2 g L \cos\vartheta = J \ddot{\vartheta}$ 3

in dobimo:

$m_u g R - m_2 g L \cos\vartheta = (J + m_u R^2)\ddot{\vartheta}$ 3

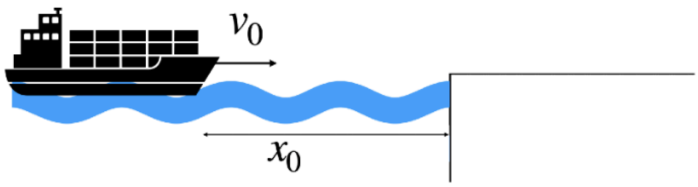
Diferencialna enačba → rešujemo z energijami!

točke so v primeru, ko se študent vseeno odloči uporabiti navore namesto energij

5. Tanker Kobayashi Maru se približuje pristanišču v Kopru z začetno hitrostjo $v_0 = 11$ m/s. Ko izključi motorje, se tankerjev pojemek spreminja kot $a = -k\,v$, kjer je $k = 2.5$ /s.

a) Po kolikem času pade hitrost tankerja na $v = 1.1$ m/s?

b) Koliko stran, x_0, od pomola mora tanker ugasniti motor, da v pomol zadane s hitrostjo manjšo kot 1.1 m/s?



a) Po kolikem času se njegova hitrost zmanjša za 10x ?

$$a = \frac{dv}{dt} = -kv \quad 3$$

$$\int_{v_0}^{v'} \frac{dv}{v} = \int_0^t -k\,dt \quad 2$$

$$\ln v = -kt$$

$$\ln v \Big|_{v_0}^{v'} = -kt$$

$$\ln\left(\frac{v'}{v_0}\right) = -kt$$

$$-\ln 10 = -kt \quad 5$$

$$t = \ln 10 / k = \underline{\underline{0.92\,s}} \quad 2$$

b) Koliko stran od pomola, mora tanker ugasniti motor, da v pomol zadane s hitrostjo manjšo od 1.1 m/s²?

$$v(t) \Rightarrow \quad \ln v \Big|_{v_0}^{v} = -kt \Big|_0^t$$

$$\ln \frac{v}{v_0} = -kt \rightarrow v = v_0 e^{-kt} \quad 3$$

$$x(t) = \int_0^t v(t)\,dt$$

$$x(t) = v_0 \int_0^t e^{-kt}\,dt = \frac{v_0}{-k} e^{-kt}\Big|_0^t = \frac{v_0}{-k}\left(e^{-kt} - 1\right) \quad 5$$

$$x(t) = \frac{v_0}{-k}\left(e^{-kt} - 1\right) \; 2 \; = \underline{3.96\text{ m}} \quad 3$$

I