Q7 — PageRank (14 points)

Part 1 (3 points) — Teleportation in Personalized PageRank

In standard PageRank, teleportation distributes the random surfer uniformly to all pages. In **personalized PageRank**, we modify the teleportation distribution to jump only to (or preferentially toward) a specific set of "topic" pages S.

This biases the ranking toward pages relevant to a particular user or topic. For example, setting S = {sports pages} produces a ranking focused on sports-related content. Different users can have different teleport sets, yielding personalized rankings from the same graph.

Part 2 (3 points) — Flaws of Outgoing-Edge-Based Importance

Two key flaws:

1. **Outgoing links are self-controlled.** Any webpage owner can add unlimited outgoing links to artificially inflate their importance score. Incoming links, by contrast, represent votes from *other* pages and cannot be manipulated by the page itself.
2. **Outgoing links don't indicate quality or popularity.** A page with many outgoing links is merely a directory or link farm — it says nothing about whether anyone finds the page valuable. Incoming links reflect that others trust and reference the page.

Part 3 (4 points) — Equal PageRank Regardless of β

Construction: A **complete directed graph** on 4 nodes — every node has an out-link to every other node.

```
  1 ↔ 2
  ↕  ╳  ↕
  3 ↔ 4
(all 12 directed edges present)
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Justification: By perfect symmetry, every node has identical in-degree (3) and out-degree (3). The transition matrix treats all nodes identically, so the unique stationary distribution is r = (1/4, 1/4, 1/4, 1/4) regardless of β. The teleportation term (1−β)/4 is also uniform, so both the random-walk component and the teleportation component distribute rank equally.

Part 4 (4 points) — Column-Stochastic Matrix

Graph edges:

- A → B, D (out-degree 2)
- B → C, D (out-degree 2)
- C → A, B, D (out-degree 3)
- D → C (out-degree 1)

Column-stochastic matrix M where $M_{ij} = 1/d_j$ if $j \to i$, else 0:

$$\boxed{M = \begin{pmatrix} 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 0 \\ 0 & 1/2 & 0 & 1 \\ 1/2 & 1/2 & 1/3 & 0 \end{pmatrix}}$$

Reading by columns:

- **Col A:** A sends 1/2 each to B and D
- **Col B:** B sends 1/2 each to C and D
- **Col C:** C sends 1/3 each to A, B, and D
- **Col D:** D sends all rank to C

Verification: Every column sums to 1 (no dead ends). ✓