Digitalna vezja
UL, FRI

Vaja 1 Booleova algebra, logisim, breadboard

Postulati Boolove algebre

Zaprtost:	**P1:** $x \vee y \in X$
	P1*: $x \cdot y \in X$
Nevtralni element:	**P2:** $x \vee 0 = x$
	P2*: $x \cdot 1 = x$
Komutativnost:	**P3:** $x \vee y = y \vee x$
	P3*: $x \cdot y = y \cdot x$
Distributivnost:	**P4:** $x \vee (y \cdot z) = (x \vee y) \cdot (x \vee z)$
	P4*: $x \cdot (y \vee z) = (x \cdot y) \vee (x \cdot z) = x \cdot y \vee x \cdot z$
Inverzni element:	**P5:** $x \vee \bar{x} = 1$
	P5*: $x \cdot \bar{x} = 0$
Število elementov:	**P6:** $x \neq y$

Lastnosti Boolove algebre

Idempotenca:

$$x \vee x \vee \ldots \vee x = x$$

$$x \cdot x \ldots \cdot x = x$$

Absorbcija:

$$x \vee (x \cdot y) = x$$

$$x \cdot (x \vee y) = x$$

Asociativnost:

$$(x \vee y) \vee z = x \vee (y \vee z) = x \vee y \vee z$$

$$(x \cdot y) \cdot z = x \cdot (y \cdot z) = x \cdot y \cdot z$$

DeMorganovo pravilo:

$$\overline{x_1 \vee x_2 \ldots \vee x_n} = \overline{x_1} \cdot \overline{x_2} \cdot \ldots \cdot \overline{x_n}$$

$$\overline{x_1 \cdot x_2 \ldots \cdot x_n} = \overline{x_1} \vee \overline{x_2} \vee \ldots \vee \overline{x_n}$$

Osnovne preklopne funkcije

Negacija (NE, NOT)

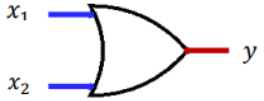
$y = \bar{x}$



x	$y = \bar{x}$
0	1
1	0

Disjunkcija (ALI, OR)

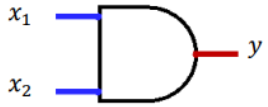
$y = x_1 \vee x_2$



x_1	x_2	$y = x_1 \vee x_2$
0	0	0
0	1	1
1	0	1
1	1	1

Konjukcija (IN, AND)

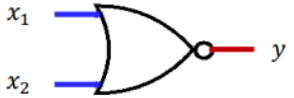
$y = x_1 \cdot x_2$



x_1	x_2	$y = x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Percipov operator (NE ALI, NOR)

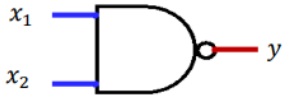
$y = x_1 \downarrow x_2 = \overline{x_1 \vee x_2}$



x_1	x_2	$y = \overline{x_1 \vee x_2}$
0	0	1
0	1	0
1	0	0
1	1	0

Shefferjerjev operator (NE IN, NAND)

$y = x_1 \uparrow x_2 = \overline{x_1 \cdot x_2}$



x_1	x_2	$y = \overline{x_1 \cdot x_2}$
0	0	1
0	1	1
1	0	1
1	1	0

Ekskluzivni ALI (XOR)

$y = x_1 \nabla x_2 = \bar{x_1} \cdot x_2 \vee x_1 \cdot \bar{x_2}$



x_1	x_2	$y = x_1 \nabla x_2$
0	0	0
0	1	1
1	0	1
1	1	0

Ekvivalenca (XNOR)

$y = x_1 \equiv x_2 = x_1 \cdot x_2 \vee \bar{x_1} \cdot \bar{x_2}$



x_1	x_2	$y = x_1 \equiv x_2$
0	0	1
0	1	0
1	0	0
1	1	1

Implikacija

$y = x_1 \rightarrow x_2 = \bar{x_1} \vee x_2$

x_1	x_2	$y = x_1 \rightarrow x_2$
0	0	1
0	1	1
1	0	0
1	1	1

Naloga 1: Poenostavljanje logičnih funkcij

- Primer: $f(x,y,z) = \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z) \vee (x \vee z)} =$

$= \overline{\bar{x} \cdot \bar{y} \vee y \cdot z} \cdot \overline{x \vee z} =$ DeMorganovo pravilo

$= \overline{\bar{x} \cdot \bar{y}} \cdot \overline{y \cdot z} \cdot \bar{x} \cdot \bar{z} =$ DeMorganovo pravilo

$= (x \vee y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{x} \cdot \bar{z} =$ DeMorganovo pravilo

$= (x \cdot \bar{x}) \vee (\bar{x} \cdot y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{z} =$ Distributivnost (P4*)

$= (0 \vee \bar{x} \cdot y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{z} =$ Inverzni element (P5*)

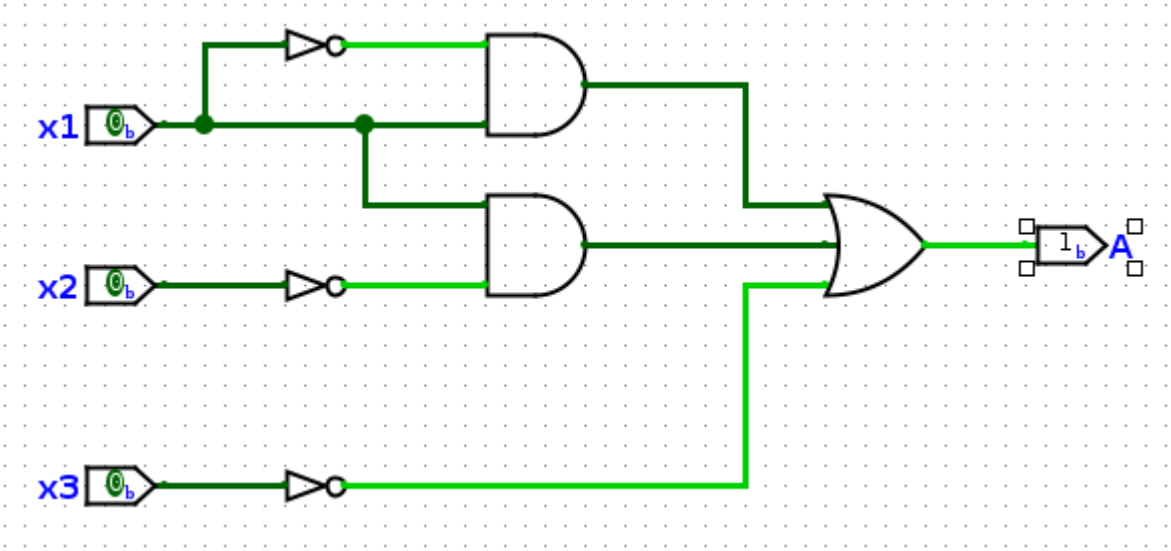
$= (\bar{x} \cdot y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{z}$ Nevtralni element (P2)

$= \bar{x} \cdot y \cdot \bar{z}$ Absorbcija

- Poenostavite logične funkcije:

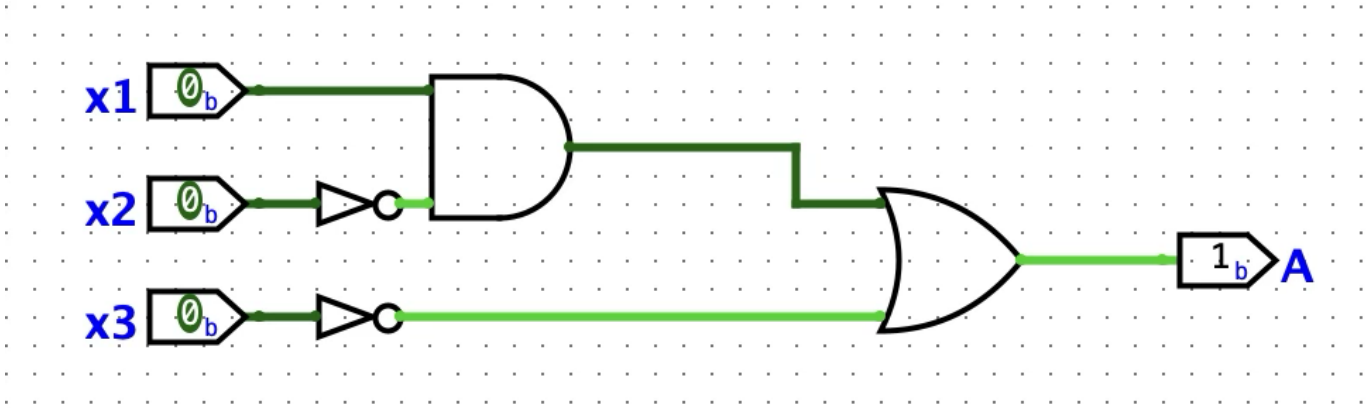
1. $f(x,y,z) = \bar{x} \cdot \bar{y} \cdot z \vee x \cdot y \cdot z \vee x \cdot y \cdot \bar{z} \vee x \cdot \bar{y} \cdot z = ?$
2. $f(x,y) = \overline{\bar{x} \cdot \bar{y} \vee x \cdot y} = ?$
3. $f(A,B,C,D) = A \vee C \cdot \overline{(\bar{A} \cdot \bar{B} \vee B)} \vee \bar{C} \cdot \overline{B \vee \bar{D}} = ?$

Naloga 2: Realizacija vezja



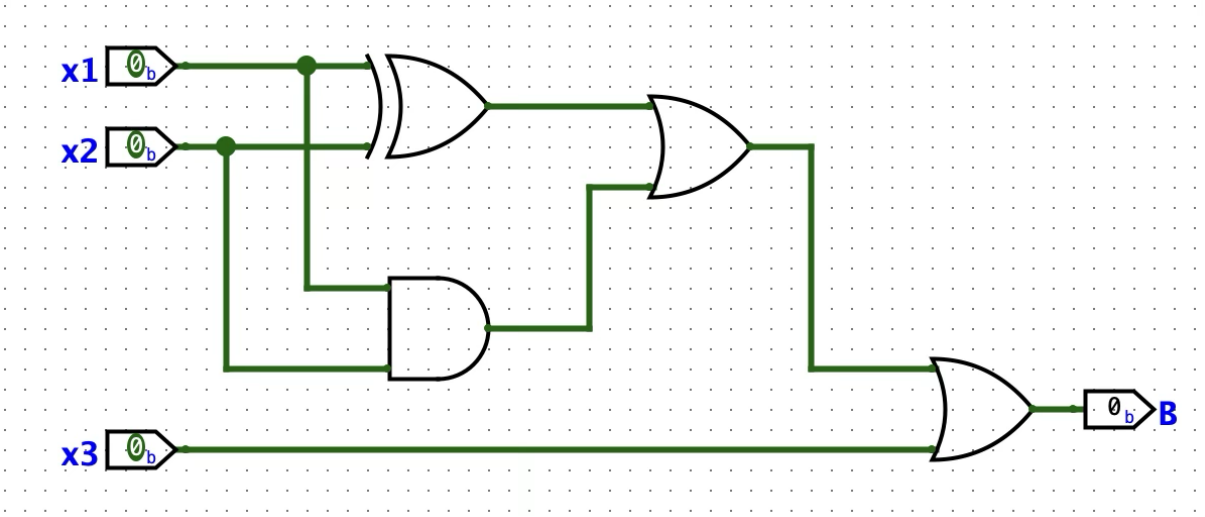
- Zapišite izhod logičnega vezja (A) z operatorji NOT, AND, OR in ga poenostavite
- Narišite vezje v Logisimu
- Vezje realizirajte na testni pološči

Naloga 2: Rešitev



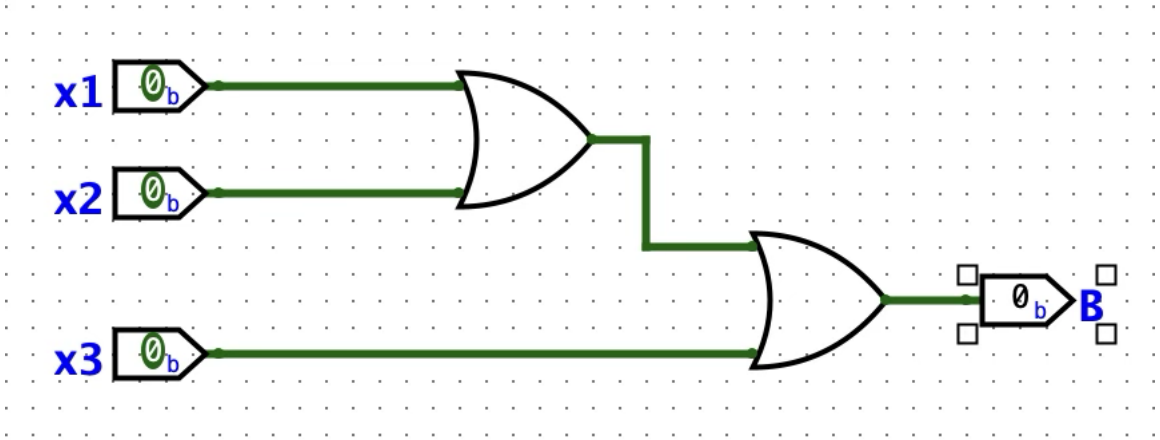
- $A = \bar{x}_1 \cdot x_1 \vee x_1 \cdot \bar{x}_2 \vee \bar{x}_3$
- Poenostavljena funkcija: $A = x_1 \cdot \bar{x}_2 \vee \bar{x}_3$

Naloga 3: Realizacija vezja



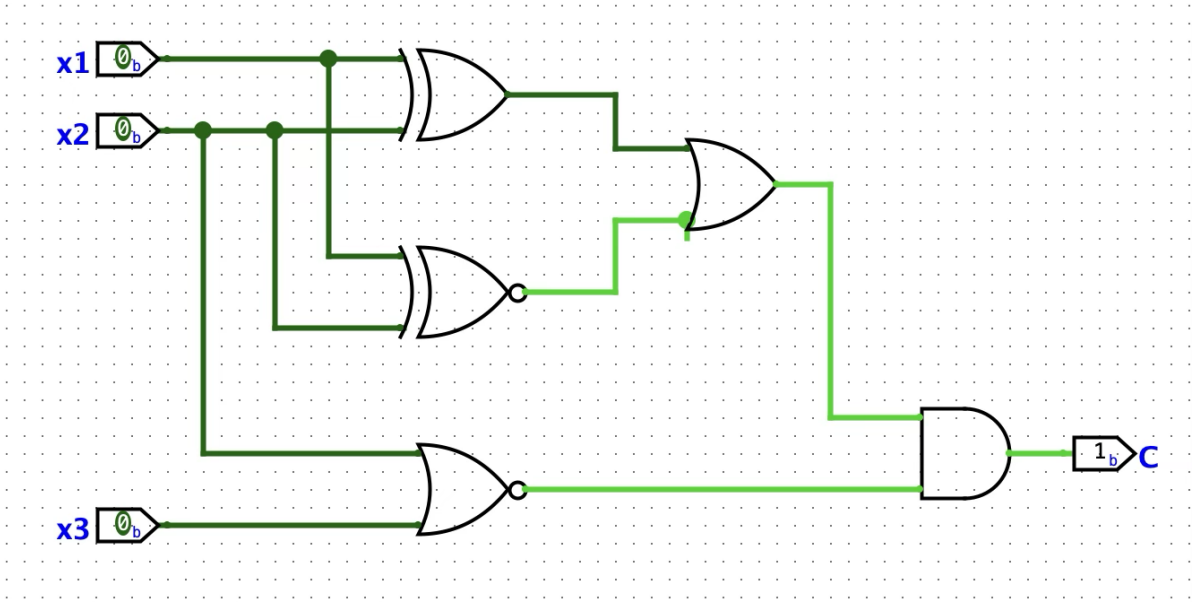
- Zapišite izhod logičnega vezja (B) z operatorji Booleove algebre in ga poenostavite
- Narišite vezje v Logisimu
- Vezje realizirajte na testni pološči

Naloga 3: Rešitev



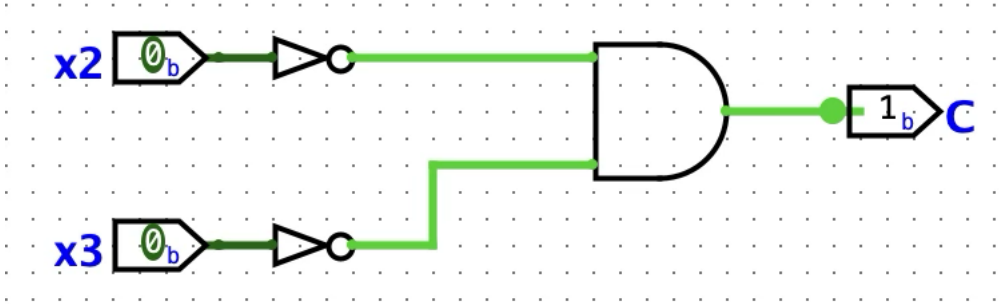
- $B = (x_1 \nabla x_2) \vee (x_1 \cdot x_2) \vee x_3$
- Poenostavljena funkcija: $B = x_1 \vee x_2 \vee x_3$

Naloga 4: Realizacija vezja



- Zapišite izhod logičnega vezja (C) z operatorji Booleove algebre in ga poenostavite
- Narišite vezje v Logisimu
- Vezje realizirajte na testni pološči

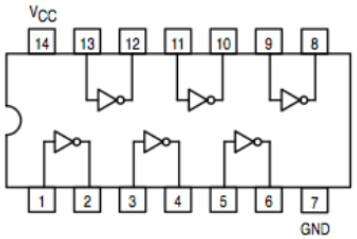
Naloga 4: Rešitev



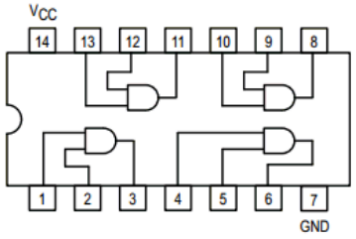
- $C = [(x_1 \nabla x_2) \vee (x_1 \equiv x_2)] \cdot (x_2 \downarrow x_3)$
- Poenostavljena funkcija: $C = \bar{x}_2 \cdot \bar{x}_3$

Testna plošča (ang. breadboard)

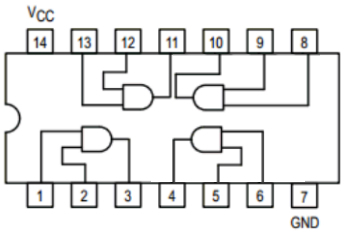
NOT (7404)



AND (7408)



AND (4081)



OR (7432)

