Computational topology
Lab work 5

1. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Vietoris-Rips complex $\mathrm{Rips}(S, R)$ for

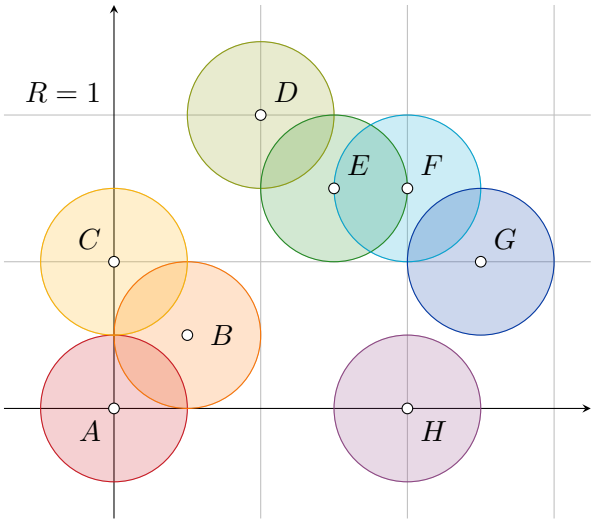
   (a) $R = 1$,

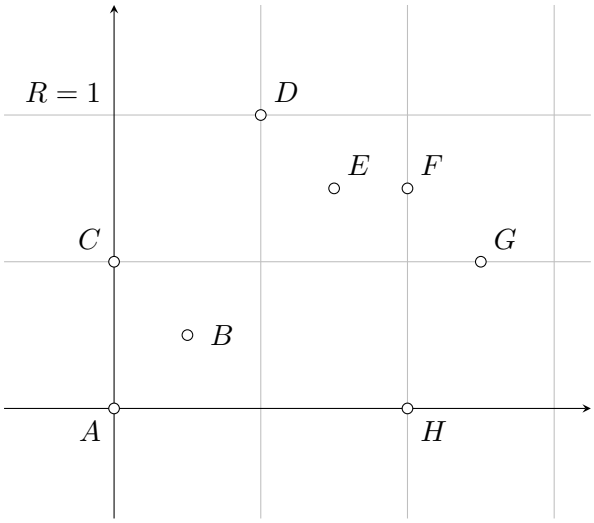
   (b) $R = 1.2$,

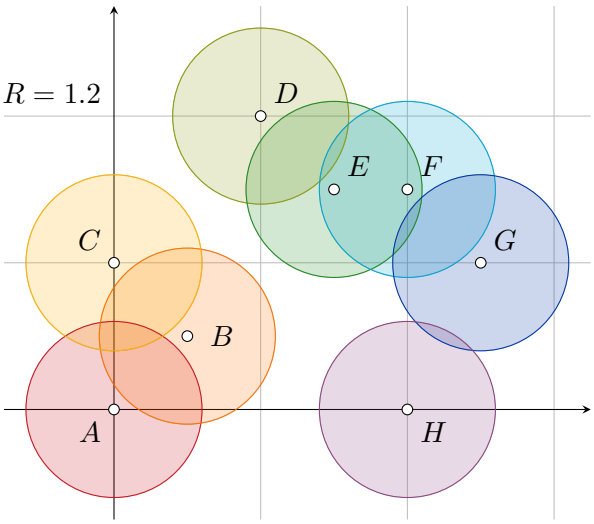
   (c) $R = 1.75$.

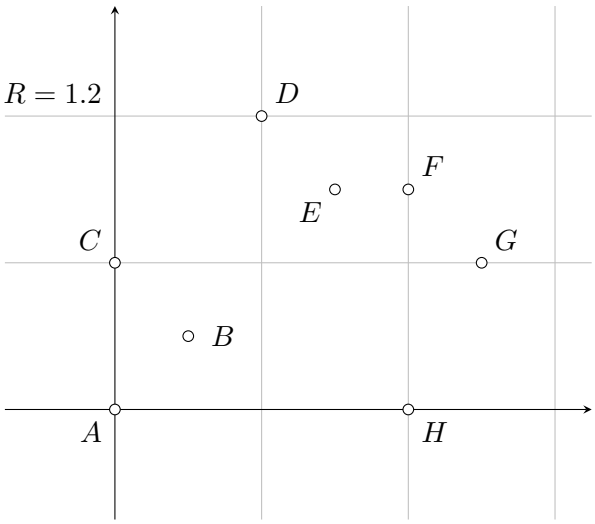
   In each case list all the simplices and determine its dimension.

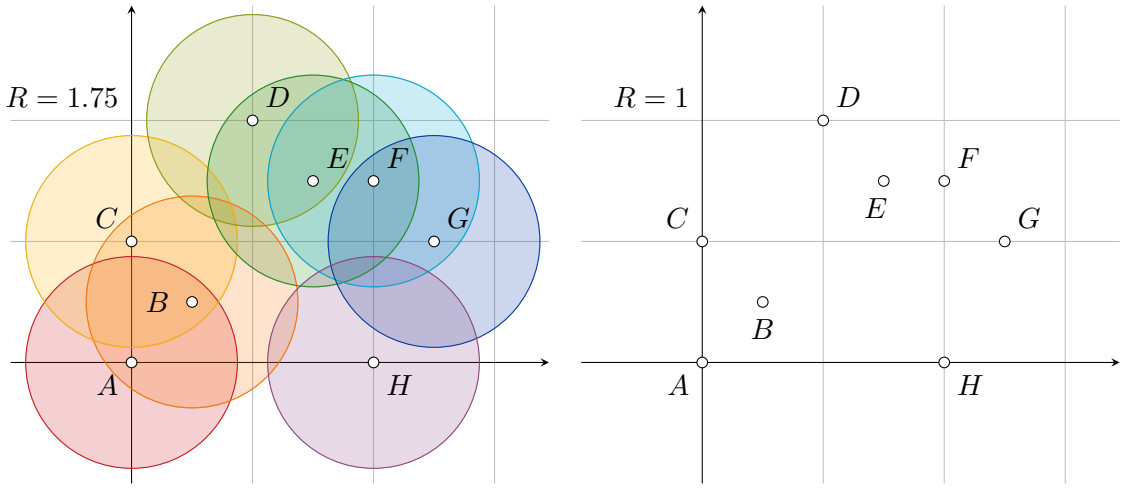
   If there is a sensor placed at each point of S and all sensors can detect points that are at distance 1.75 or less, is the area covered by the sensors connected? Does it contain any holes?











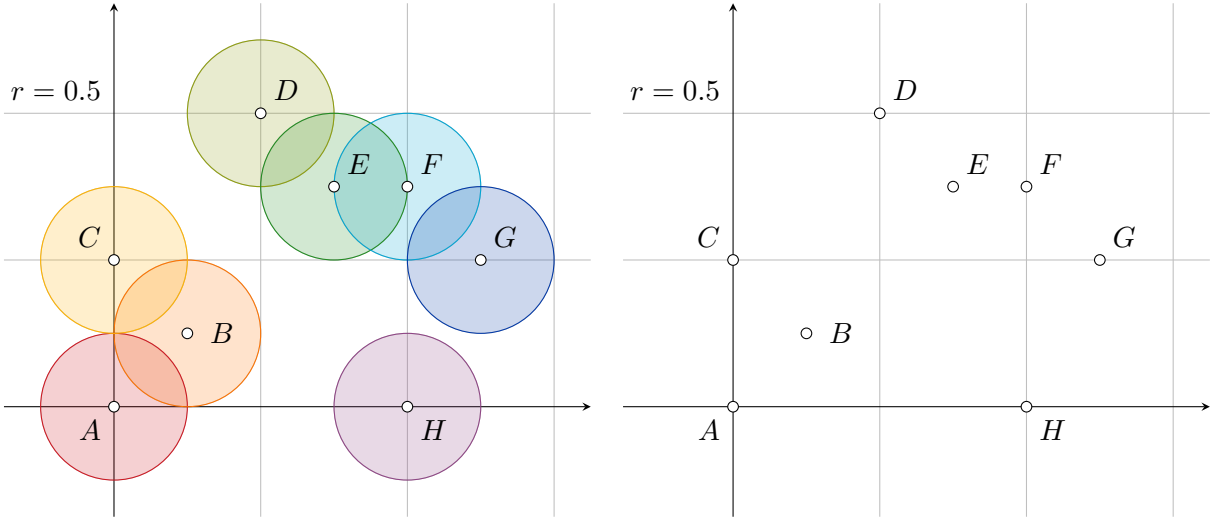
2. Let $S = \{A(0,0), B(0.5,0.5), C(0,1), D(1,2), E(1.5,1.5), F(2,1.5), G(2.5,1), H(2,0)\} \subset \mathbb{R}^2$. Build the Čech complex $\text{Cech}(S,r)$ for

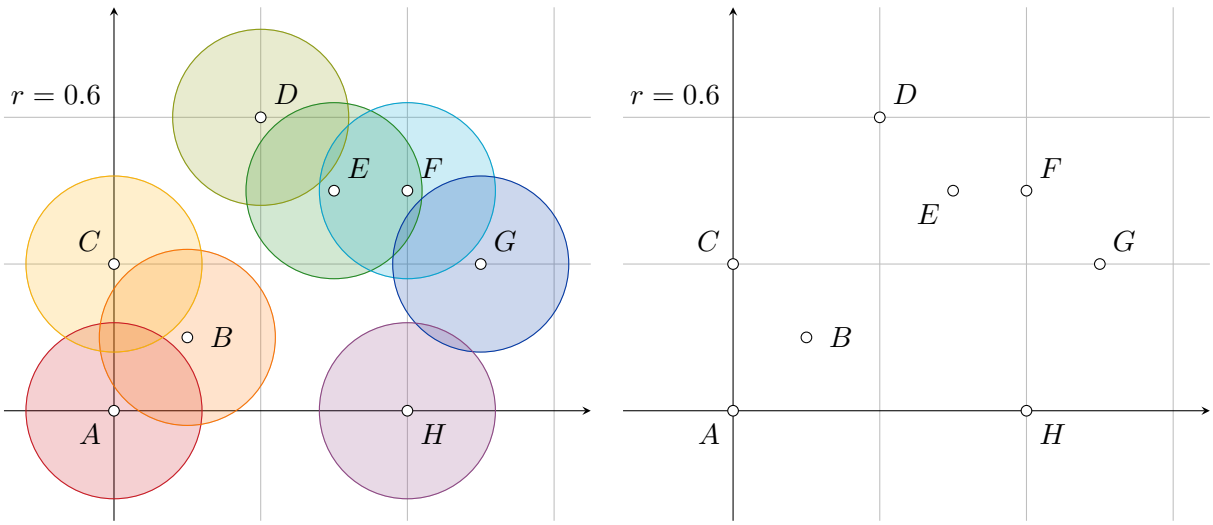
   (a) $R = 0.5$,

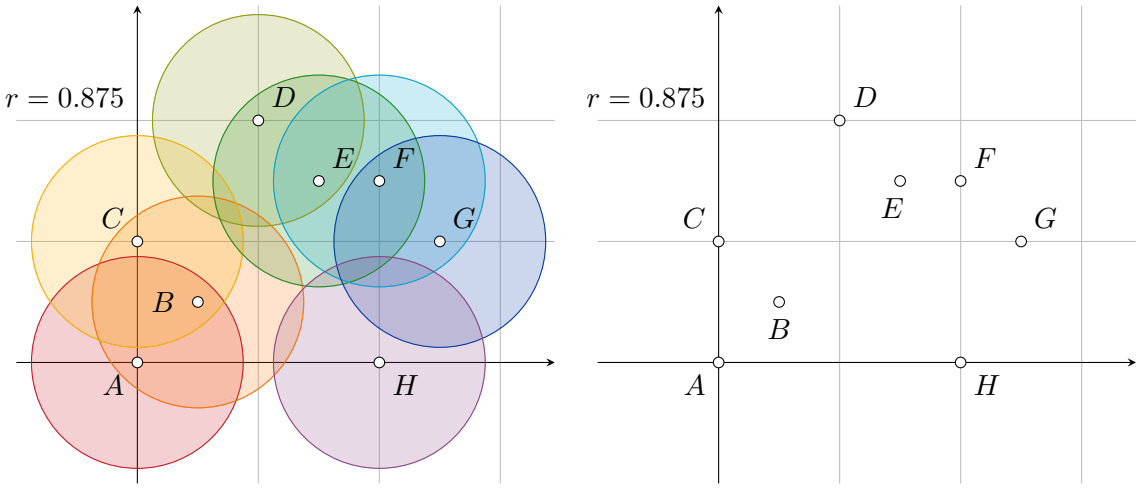
   (b) $R = 0.6$,

   (c) $R = 0.875$.

   In each case list all the simplices and determine its dimension.







3. Using the Levenshtein distance on the set of words

$$S = \{\texttt{SONCE}, \texttt{SENCE}, \texttt{SREDA}, \texttt{VRAG}, \texttt{SRENJ}\},$$

build the filtration of the Vietoris-Rips complexes

$$\mathrm{Rips}(S,1) \subseteq \mathrm{Rips}(S,2) \subseteq \mathrm{Rips}(S,3) \subseteq \mathrm{Rips}(S,4) \subseteq \mathrm{Rips}(S,5) \subseteq \mathrm{Rips}(S,6).$$

To speed things up you can use an online Levenshtein distance calculator or write your own version.

4. Given the following triangulations of the cylinder X and the Moebius band Y, find a sequence of elementary collapses that simplifies them as much as possible.

