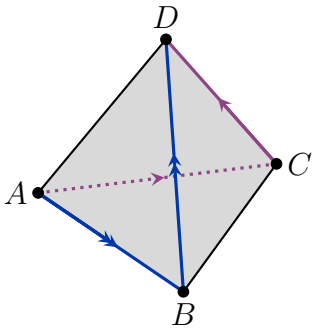
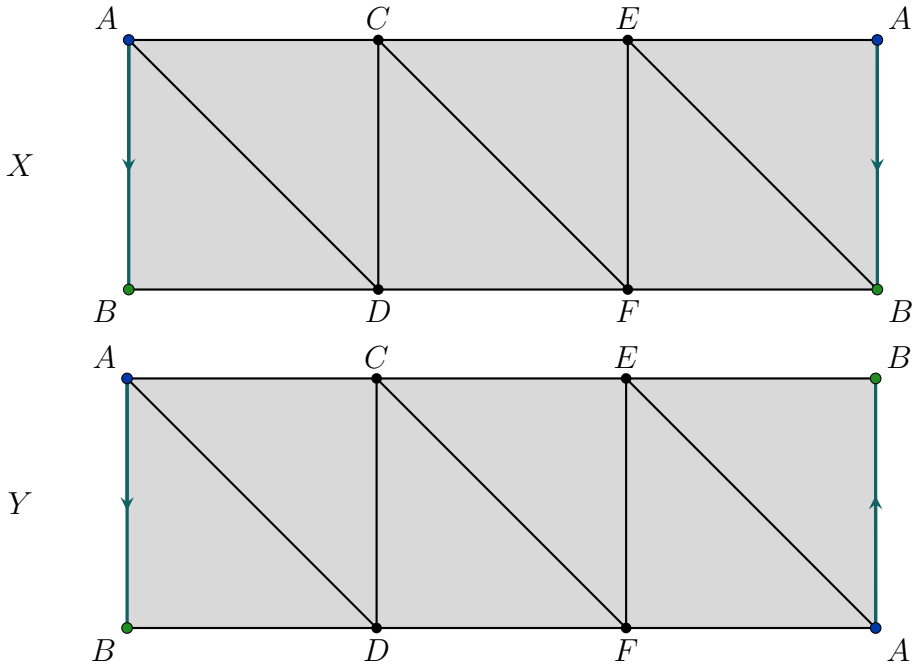
Computational topology
Lab work 6

1. Let $X = \Delta^3$ be the standard 3-simplex (tetrahedron) with vertices A, B, C and D. We obtain Y by identifying the edges AB and BD and the edges AC and CD (preserving the ordering of vertices). Show that Y collapses onto a Klein bottle.



2. Given the following triangulations of the cylinder X and the Moebius band Y, find a sequence of elementary collapses that simplifies them as much as possible, then compute the homology groups $H_*(X)$ and $H_*(Y)$.



3. For the simplicial complex X in the figure below

   (a) write down the chain groups $\mathcal{C}_n$,

   (b) determine the boundary homomorphisms $\partial_n \colon \mathcal{C}_n \to \mathcal{C}_{n-1}$,

   (c) find the cycles $Z_n = \ker\partial_n$,

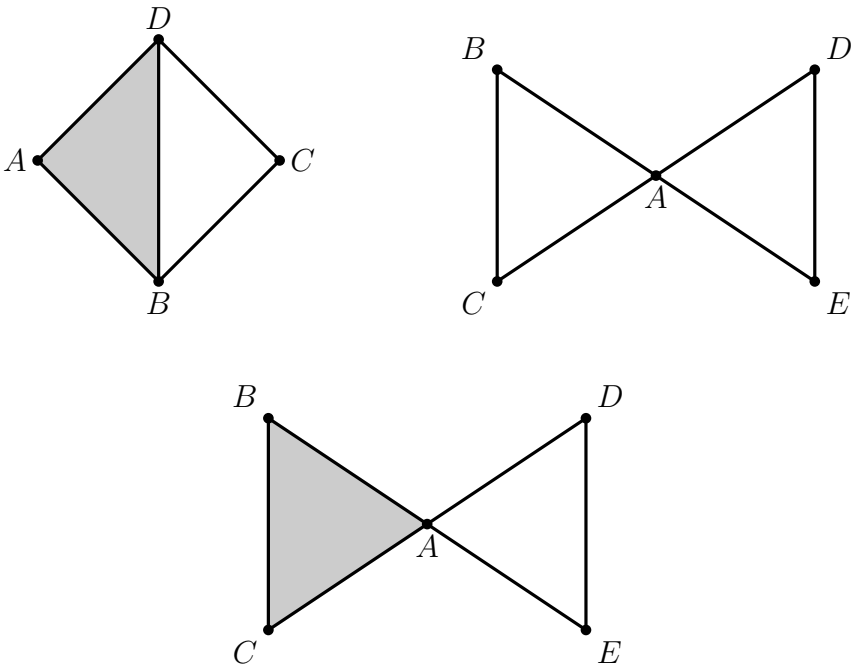
   (d) find the boundaries $B_n = \mathrm{im}\partial_n$,

   (e) determine the simplicial homology groups with $\mathbb{Z}$ coefficients, $H_n(X;\mathbb{Z})$,

   (f) determine the simplicial homology groups with $\mathbb{Z}_2$ coefficients, $H_n(X;\mathbb{Z}_2)$,

(g) determine the Betti numbers of X and

(h) compute the Euler characteristic of X.



4. Let X be a simplicial complex with maximal simplices ABC, ABD, ACD, BCD, BE and CE. Compute $H_*(X)$.

