Computational Complexity and Heuristic Programming, 2017/18
Written exam, 9 February 2017

All questions count equally. Literature, electronic and communication devices are not allowed. It is allowed to use up to 2 sheets of A4 format paper with handwritten notes. You can write your answers in Slovene. Duration: 90 minutes.
Oral exam for students who wish to improve their grade and have achieved at least 50% of points in written exam, will take place on Wednesday, 14 February 2018 at 13:00, in the office of Prof Robnik Šikonja (2nd floor, room 2.06).

1. Find the solution to the recurrence
   $T(n) = 5T(n-1) - 8T(n-2) + 4T(n-3)$
   with base cases $T(0) = 1$ and $T(1) = T(2) = 0$.

2. We have the following randomized algorithm for computing the smallest element in an array
   ```
   1 int randomMin(A[1 .. n]) {
   2    int min = ∞
   3    for int i = 1 to n in random order
   4       if A[i] < min
   5          min = A[i]
   6    return min
   7 }
   ```
   What is the probability that line 5 is executed during the *n*-th iteration of the for loop? What is the exact expected number of executions of line 5?

3. In a network *G = (V, E)* we are given for each edge $(u, v) \in E$ a capacity *c(u, v)* and edge cost *a(u, v)*. We assume that *c(u, v)* = 0 if edge (u, v) is not in *E* and that there are no antiparallel edges. If we send f_{uv} units of flow over edge *(u, v)* we incur a cost of $a(u,v)f_{uv}$. Note that we are not given a source, sink or demand. We consider a flow feasible if it satisfies the capacity constraint on every edge and flow conservation at every vertex. The goal is to find, among all feasible flows, the one of minimum cost.
   a) Formulate the above stated problem as a linear program.
   b) Assume that for all edges $(u, v) \in E$ we have costs $a(u, v) \geq 0$. Characterize an optimal solution to the above stated minimum-cost feasible flow.

4. The minimum set-difference problem takes as input a set S of numbers and tries to partition it into two mutually exclusive subsets A and B ($A \cup B = S$ and $A \cap B = \emptyset$), such that the difference of sums is as small as possible: $\min|\sum_{x \in A} x - \sum_{x \in B} x|$. Justify your answers to the following problems.
   a) Propose a data structure representing a solution of this problem that could be used in local optimization. Write a pseudocode of neighbourhood generator.
   b) Propose features, which could be used in penalization with guided local search. Justify your proposal.
   c) Suggest contents of tabu lists, which could be useful in tabu search.