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## N-gram language models



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## Contents

- language models
- $n$-grams (still used in the evaluation measures)
mostly based on Jurafsky \& Martin, $3^{\text {rd }}$ edition, read Chapter 3.1-3.4


## Probabilistic Language Models

-The goal: assign a probability to a sentence

- Machine Translation:
- P (high winds tonight) $>\mathrm{P}$ (large winds tonight)
- Spell Correction

Why?

- The office is about fifteen minuets from my house
- P(about fifteen minutes from) > P(about fifteen minuets from)
- Speech Recognition
- P(I saw a van) >> P(eyes awe of an)
-     + Summarization, question-answering, etc., etc.!!


## Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

$$
P(W)=P\left(w_{1}, w_{2}, w_{3}, w_{4}, w_{5} \ldots w_{n}\right)
$$

- Related task: probability of an upcoming word:

$$
\mathrm{P}\left(\mathrm{w}_{5} \mid \mathrm{w}_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}, \mathrm{w}_{4}\right)
$$

- A model that computes either of these:
$\mathrm{P}(\mathrm{W})$ or $\mathrm{P}\left(\mathrm{w}_{\mathrm{n}} \mid \mathrm{w}_{1}, \mathrm{w}_{2} \ldots \mathrm{w}_{\mathrm{n}-1}\right)$ is called a language model.
- A better name would be: the grammar model
- But language model or LM is standard


## How to statistically compute $\mathrm{P}(\mathrm{W})$

- How to compute this joint probability:

$$
P(\text { its, water, is, so, transparent, that) }
$$

- Intuition: let's rely on the Chain Rule of Probability


## Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$
\mathrm{p}(\mathrm{~B} \mid \mathrm{A})=\mathbf{P}(\mathbf{A}, \mathrm{B}) / \mathbf{P}(\mathrm{A}) \quad \text { Rewriting: } \mathbf{P}(\mathbf{A}, \mathrm{B})=\mathbf{P}(\mathbf{A}) \mathbf{P}(\mathrm{B} \mid \mathrm{A})
$$

- More variables:

$$
P(A, B, C, D)=P(A) P(B \mid A) P(C \mid A, B) P(D \mid A, B, C)
$$

-The Chain Rule in General

$$
P\left(x_{1}, x_{2}, x_{3}, \ldots, x_{n}\right)=P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3} \mid x_{1}, x_{2}\right) \ldots P\left(x_{n} \mid x_{1}, \ldots, x_{n-1}\right)
$$

The Chain Rule applied to compute joint probability of words in sentence

$$
P\left(w_{1} w_{2} \ldots w_{n}\right)=\prod_{i} P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right)
$$

P ("its water is so transparent") $=$ $P($ its $) \times P($ water $\mid$ its $) \times P($ is $\mid$ its water $)$
$\times \mathrm{P}$ (solits water is) $\times \mathrm{P}$ (transparent $\mid$ its water is so)

## How to estimate these probabilities

- Could we just count and divide?
$P($ the $\mid$ its water is so transparent that $)=$ $\underline{\text { Count (its water is so transparent that the) }}$
Count(its water is so transparent that)
- No! Too many possible sentences!
- We'll never see enough data for estimating these


## Markov Assumption

- The memory is short
- First order Markov assumption

Andrei Markov
$P$ (the |its water is so transparent that) $\quad P$ (the |that)

- The second order Markov assumption
$P$ (the |its water is so transparent that) $\quad P$ (the |transparent that)

Using Markov assumption of order k

$$
P\left(w_{1} w_{2} \ldots w_{n}\right) \approx \prod_{i} P\left(w_{i} \mid w_{i-k} \ldots w_{i-1}\right)
$$

- In other words, we approximate each component in the product

$$
P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right) \approx P\left(w_{i} \mid w_{i-k} \ldots w_{i-1}\right)
$$

## Simplest case: Unigram model

$$
P\left(w_{1} w_{2} \ldots w_{n}\right) \approx \prod_{i} P\left(w_{i}\right)
$$

Some automatically generated sentences from a unigram model
fifth, an, of, futures, the, an, incorporated, a, a, the, inflation, most, dollars, quarter, in, is, mass
thrift, did, eighty, said, hard, 'm, july, bullish
that, or, limited, the

## Bigram model

- Condition on the previous word:


## $P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right) \approx P\left(w_{i} \mid w_{i-1}\right)$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen
outside, new, car, parking, lot, of, the, agreement, reached
this, would, be, a, record, november

## Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$
\begin{gathered}
P\left(w_{i} \mid w_{i 1}\right)=\frac{\operatorname{count}\left(w_{i 1}, w_{i}\right)}{\operatorname{count}\left(w_{i 1}\right)} \\
P\left(w_{i} \mid w_{i 1}\right)=\frac{c\left(w_{i 1}, w_{i}\right)}{c\left(w_{i 1}\right)}
\end{gathered}
$$

## An example

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$$
P\left(w_{i} \mid w_{i 1}\right)=\frac{c\left(w_{i 1}, w_{i}\right)}{c\left(w_{i 1}\right)}
$$

$$
\begin{array}{lll}
P(\mathrm{I}|<\mathrm{s}\rangle)=\frac{2}{3}=.67 & P(\mathrm{Sam} \mid\langle\mathrm{s}\rangle)=\frac{1}{3}=.33 & P(\mathrm{am} \mid \mathrm{I})=\frac{2}{3}=.67 \\
P(</ \mathrm{s}\rangle \mid \mathrm{Sam})=\frac{1}{2}=0.5 & P(\mathrm{Sam} \mid \mathrm{am})=\frac{1}{2}=.5 & P(\mathrm{do} \mid \mathrm{I})=\frac{1}{3}=.33
\end{array}
$$

N -gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
- because language has long-distance dependencies:
"The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing."
- N -gram models are better in English than in Slovene and many other languages. Why?


## Example: Restaurant sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day


## Raw bigram counts

- Out of 9222 sentences

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

## Raw bigram probabilities

- Normalize by unigrams:
- Result:

| i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2533 | 927 | 2417 | 746 | 158 | 1093 | 341 | 278 |


|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 0.002 | 0.33 | 0 | 0.0036 | 0 | 0 | 0 | 0.00079 |
| want | 0.0022 | 0 | 0.66 | 0.0011 | 0.0065 | 0.0065 | 0.0054 | 0.0011 |
| to | 0.00083 | 0 | 0.0017 | 0.28 | 0.00083 | 0 | 0.0025 | 0.087 |
| eat | 0 | 0 | 0.0027 | 0 | 0.021 | 0.0027 | 0.056 | 0 |
| chinese | 0.0063 | 0 | 0 | 0 | 0 | 0.52 | 0.0063 | 0 |
| food | 0.014 | 0 | 0.014 | 0 | 0.00092 | 0.0037 | 0 | 0 |
| lunch | 0.0059 | 0 | 0 | 0 | 0 | 0.0029 | 0 | 0 |
| spend | 0.0036 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 |

## Bigram estimates of sentence probabilities

$\mathrm{P}(\langle\mathrm{s}\rangle$ | want english food $</ \mathrm{s}\rangle$ ) = $\mathrm{P}(\mathrm{I} \mid<\mathrm{s}>)$
$\times P($ want $\mid I)$
$\times \mathrm{P}($ english|want)
$\times \mathrm{P}$ (food $\mid$ english)
$\times \mathrm{P}(</ \mathrm{s}>\mid$ food $)$
= . 000031

What kinds of knowledge bigram LM contains?
$\bullet P($ english|want) $=.0011$
$\bullet P($ chinese $\mid$ want $)=.0065$

- $P($ to| want $)=.66$
$-P($ eat $\mid$ to $)=.28$
- P(food | to) = 0
- $P($ want $\mid$ spend $)=0$
- $P(i \mid<s>)=.25$

Practical Issues

- We do everything in log space instead of probability space
- Avoids underflow
- (also adding is faster than multiplying)
$\log \left(p_{1} \quad p_{2} \quad p_{3} \quad p_{4}\right)=\log p_{1}+\log p_{2}+\log p_{3}+\log p_{4}$


## Google Book N-Grams, 2006

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are $13,588,391$ unique words, after discarding words that appear less than 200 times.

## Google N-Gram Release

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensible 40
- serve as the individual 234


## Language Modeling Tools

- are ngram language models still useful?
- yes, e.g., in speech processing
- mostly replaced by neural LMs
- many variants of adapted neural LMs exist, e.g., word2vec, fastText, ELMo, BERT


## Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
- Assign higher probability to "real" or "frequently observed" sentences
- Than "ungrammatical" or "rarely observed" sentences?
- We train parameters of our model on a training set.
- We test the model's performance on data we haven't seen.
- A test set is an unseen dataset that is different from our training set, totally unused.
- An evaluation metric tells us how well our model does on the test set.
- Two types of evaluation
- intrinsic (internal)
- extrinsic (external, on a downstream task)


## Extrinsic evaluation of N -gram models

- Best evaluation for comparing models $A$ and $B$
- Use each model in a task
- spelling corrector, speech recognizer, MT system
- Run the task, get an accuracy for $A$ and for $B$
- How many misspelled words corrected properly
- How many words translated correctly
- Compare accuracy for $A$ and $B$


## Difficulty of extrinsic (in-vivo) evaluation of N -gram models

- Extrinsic evaluation
- Time-consuming; can take days or weeks
- So
- Sometimes use intrinsic evaluation: perplexity
- Bad approximation
- unless the test data looks just like the training data
- So generally only useful in pilot experiments
- But is helpful to think about.


## Intuition of Perplexity

- The Shannon Game:
- How well can we predict the next word?

I always order pizza with cheese and
The $33^{\text {rd }}$ President of the US was $\qquad$
I saw a $\qquad$ _

- Unigrams are terrible at this game. (Why?)
mushrooms 0.1 pepperoni 0.1
anchovies 0.01
....
fried rice 0.0001
....
and 1e-100
- A better model of a text
- is one which assigns a higher probability to the word that actually occurs


## Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of

$$
P P(W)=P\left(w_{1} w_{2} \ldots w_{N}\right)^{\frac{1}{N}}
$$ the test set, normalized by the number of words:

$$
=\sqrt[N]{\frac{1}{P\left(w_{1} w_{2} \ldots w_{N}\right)}}
$$

Chain rule:

$$
\begin{aligned}
& \operatorname{PP}(W)=\sqrt[N]{\prod_{i=1}^{N} \frac{1}{P\left(w_{i} \mid w_{1} \ldots w_{i-1}\right)}} \\
& \operatorname{PP}(W)=\sqrt[N]{\prod_{i=1}^{N} \frac{1}{P\left(w_{i} \mid w_{i-1}\right)}}
\end{aligned}
$$

Minimizing perplexity is the same as maximizing probability

## Perplexity example

- Let us suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign $\mathrm{P}=1 / 10$ to each digit?

$$
\begin{aligned}
\operatorname{PP}(W) & =P\left(w_{1} w_{2} \ldots w_{N}\right)^{-\frac{1}{N}} \\
& =\left(\frac{1}{10}^{N}\right)^{-\frac{1}{N}} \\
& =\frac{1}{10}^{-1} \\
& =10
\end{aligned}
$$

## Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ


## N-gram Unigram $\quad$ Bigram Trigram Order <br> Perplexity 962 <br> 170 <br> 109

## The Shannon Visualization Method

- Choose a random bigram
(<s>, w) according to its probability
- Now choose a random bigram ( $w, x$ ) according to its probability
- And so on until we choose </s>

```
<s> I 
    want to
        to eat
            eat Chinese
                Chinese food
                                    food </s>
```

- Then string the words together $I_{\text {I }}$ want to eat Chinese food


## Approximating Shakespeare

-To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
-Hill he late speaks; or! a more to leg less first you enter
-Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
-What means, sir. I confess she? then all sorts, he is trim, captain.
-Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.
-This shall forbid it should be branded, if renown made it empty.
-King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
gram -It cannot be but so.

## Shakespeare as corpus

- $\mathrm{N}=884,647$ tokens, $|\mathrm{V}|=29,066$
- Shakespeare produced 300,000 bigram types out of $|\mathrm{V}|^{2}=844$ million possible bigrams.
- So $99.96 \%$ of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams are even worse: What's coming out looks like Shakespeare because it is Shakespeare


## The Wall Street Journal

1gat

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Last December through the way to preserve the Hudson corporation N.
B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

## What is the source of these random 3-gram sentences?

- They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and gram Brazil on market conditions
- This shall forbid it should be branded, if renown made it empty.
- "You are uniformly charming!" cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.


## The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
- In real life, it often doesn't
- We need to train robust models that generalize!
- One kind of (outdated) generalization: Zeros!
- Things that don't ever occur in the training set
- But occur in the test set
- In practice, we use neural language models

