I. (a) Denote by $T(t)$ the temperature of the beer can at time $t$, let $T_0 = T(0)$ be the initial temperature of the can, and let $T_F$ be the constant temperature inside the fridge.

\[
\frac{dT}{dt} \propto \text{change in temperature is proportional to } T - T_F, \text{ i.e.}
\]

\[
\frac{dT}{dt} = k(T - T_F)
\]

(b) We need to solve this DE:

\[
\frac{dT}{dt} = k(T - T_F) \
\Rightarrow \frac{dT}{T - T_F} = k \, dt
\]

Hence:

\[
\log(T - T_F) = kt + C',
\]

therefore:

\[
T - T_F = e^{kt} \frac{e^{C'}}{C} \quad \text{or} \quad T(t) = C e^{kt} + T_F.
\]

Plugging in our initial condition $T(0) = T_0$ we get:

\[
T(0) = C + T_F = T_0 \quad \text{or} \quad C = T_0 - T_F.
\]

and finally:

\[
T(t) = (T_0 - T_F) e^{kt} + T_F.
\]

In our particular case $T_0 = 24$, $T_F = 4$ and $T(1/2) = 14$ (if we measure the time in hours), i.e.

\[
14 = 20 e^{-k/2} + 4 \quad \text{or} \quad \frac{4}{2} = e^{k/2} \quad \text{or} \quad k = -2 \log 2.
\]

To cool it down to 9°C:

\[
g = (24 - 4) e^{-2 \log 2 \cdot t} + 4 \quad \text{or} \quad \frac{4}{4} = e^{-2 \log 2 \cdot t} \quad \text{at } t = 1,
\]

ie. Victor needs to keep the beer can in the fridge for 1 hour to cool it down to 9°C.
2. The DE \( y' = 2x(y^2 + 1) \) has separable variables, i.e. \( \frac{dy}{dx} = 2x(y^2 + 1) \div \frac{dx}{y^2 + 1} \) gives \( \int \frac{dy}{y^2 + 1} = \int 2x \, dx \), integrating both sides we get:

\[
\int \frac{dy}{y^2 + 1} = \int 2x \, dx \quad \text{arctan} y = x^2 + C,
\]

hence \( y = \tan(x^2 + C) \) is the general solution.

To solve the initial value problem we plug the initial condition \( y(1) = 0 \) into our general solution:

\[
0 = \tan(1^2 + C) \quad 1 + C = 0 \quad C = -1.
\]

The solution to the given initial value problem is therefore \( y = \tan(x^2 - 1) \).

3. Again, the equation \( y' = cy \cdot \left(1 - \frac{y}{a}\right) \) has separable variables:

\[
\frac{dy}{dx} = cy \cdot \left(1 - \frac{y}{a}\right) \div \frac{dx}{y \cdot \left(1 - \frac{y}{a}\right)} \quad \int \frac{dy}{y \cdot \left(1 - \frac{y}{a}\right)} = c \int dx.
\]

Now, \( \frac{1}{y \cdot \left(1 - \frac{y}{a}\right)} = \frac{1}{y} - \frac{1}{y - a} \) and:

\[
\int \left(\frac{1}{y} - \frac{1}{y - a}\right) \, dy = \int c \, dx \quad \log \frac{y}{y - a} = cx + \log C.
\]

Hence, \( \frac{y}{y - a} = Ce^{cx} \) or \( 1 - \frac{a}{y} = \frac{e^{-cx}}{C} \)

\[
\frac{a}{y} = 1 - \frac{e^{-cx}}{C} \quad y = \frac{aC}{C - e^{-cx}} \quad \text{general solution}
\]

With the initial condition \( y(0) = b \) we get:

\[
b = \frac{aC}{C - 1} \quad \frac{1}{C} = 1 - \frac{a}{b} \quad C = \frac{b}{b - a},
\]

and \( y(x) = \frac{ab}{b - (a - b)e^{-cx}} \).