1. See 4^{th} exercise, week 10.

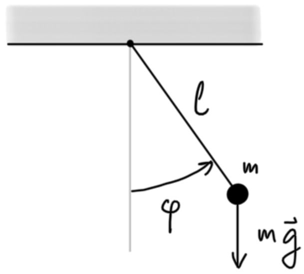
2. (a) $\dot{S}+\dot{I}+\dot{R} = -\cancel{\frac{\beta IS}{N}} + \cancel{\frac{\beta IS}{N}} - \cancel{\gamma I} + \cancel{\gamma I} = 0$, hence

(since the sum of derivatives is the derivative of the sum)

$$\dot{S}+\dot{I}+\dot{R} = (S+I+R)' = 0 \Rightarrow S+I+R = \text{const},$$

(b) $S+I+R = 1$ ... $R = 1-S-I$.

3. (a) We'll use Newton's 2^{nd} law for circular motion:



$$\vec{\tau} = J\vec{\alpha} \quad (\text{instead of } \vec{F} = m\vec{a})$$

torque angular acceleration

Since $\alpha = \ddot{\varphi}$, $J = m\ell^2$, and $\tau = \ell \cdot (-mg \sin\varphi)$, we get:

$$-mg\ell \sin\varphi = m\ell^2 \ddot{\varphi} \quad \dots \quad \boxed{\ddot{\varphi} + \frac{g}{\ell}\sin\varphi = 0}.$$

(b) From $\ddot{\varphi} = -\frac{g}{\ell}\sin\varphi$ (and $\omega = \dot{\varphi}$) we get:

$$\boxed{\begin{aligned} \dot{\varphi} &= \omega \\ \dot{\omega} &= -\frac{g}{\ell}\sin\varphi \end{aligned}} \quad (\text{since } \ddot{\varphi} = \dot{\omega}).$$