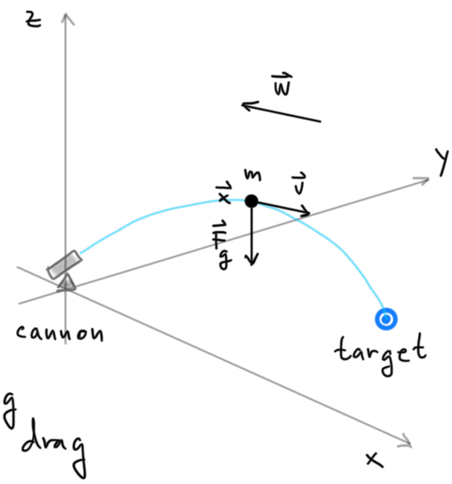
1. (a)




Forces acting upon the projectile:

- gravity: $\vec{F}_g = m\vec{g}$, where $\vec{g} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$,
- air drag: $\vec{F}_d = C \cdot \|\vec{v} - \vec{w}\|^2 \cdot \left( - \dfrac{\vec{v} - \vec{w}}{\|\vec{v} - \vec{w}\|} \right)$,

where $C$ is a constant depending on the mass, cross section, and drag coefficient.

Hence, from Newton's $2^{nd}$ law $\vec{F} = m\vec{a}$, we get:

$$m\ddot{\vec{x}} = \vec{F}_g + \vec{F}_d \quad \ldots \quad m\ddot{\vec{x}} = m\vec{g} + \underset{mc}{\underset{\parallel}{C}} \, (\vec{w} - \underset{\dot{\vec{x}}}{\underset{\parallel}{\vec{v}}}) \, \|\vec{w} - \underset{\dot{\vec{x}}}{\underset{\parallel}{\vec{v}}}\| \quad \ldots$$

$$\ldots \quad \boxed{\ddot{\vec{x}} = \vec{g} + c(\vec{w} - \dot{\vec{x}})\|\vec{w} - \dot{\vec{x}}\|}.$$

(b) From $\dot{\vec{x}} = \vec{v}$ (which we already used) we get:

$$\boxed{\begin{aligned} \dot{\vec{x}} &= \vec{v}, \\ \dot{\vec{v}} &= \vec{g} + c\,(\vec{w} - \vec{v})\|\vec{w} - \vec{v}\|. \end{aligned}}$$

(c) Using (b) we will write

$$\boxed{\vec{X}(t) = \begin{bmatrix} \vec{x}(t) \\ \vec{v}(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix}}$$

as our 'position' vector for the $4^{th}$ order Runge-Kutta method.

The initial condition is:

$$\boxed{\vec{X}(0) = \begin{bmatrix} \vec{x}(0) \\ \vec{v}(0) \end{bmatrix} = \begin{bmatrix} \vec{0} \\ \vec{v}_0 \end{bmatrix} = v_0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cos\varphi \cos\vartheta \\ \sin\varphi \cos\vartheta \\ \sin\vartheta \end{bmatrix}},$$

where $v_0$ is a constant (initial speed of the projectile at the moment of firing) and angles $\varphi$ and $\vartheta$ determine the orientation of the barrel.