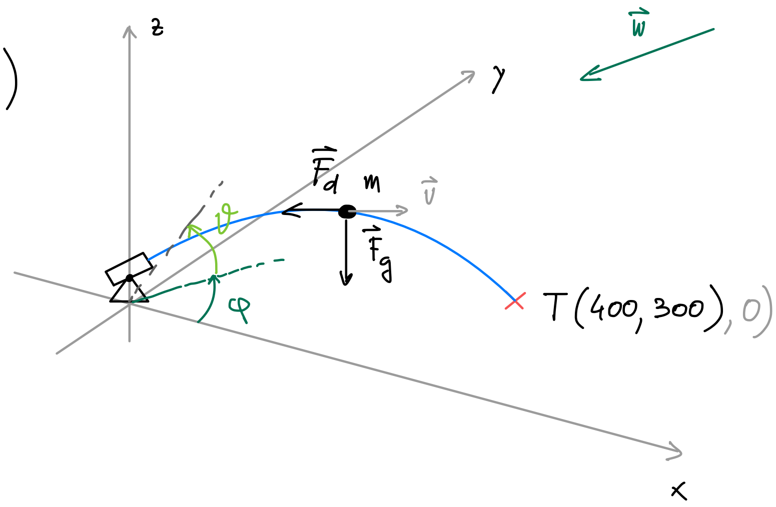
Matematično modeliranje (FRI), vaje, 18. 05. 2021

(a)



$$\vec{F} = m\vec{a}$$

$$\vec{F} = \vec{F}_g + \vec{F}_d =$$

drag coefficient

$$m\vec{a} = m\vec{g} + c'(\vec{w} - \vec{v})\|\vec{w} - \vec{v}\|$$

$$\vec{v} = \dot{\vec{x}}, \quad \vec{a} = \dot{\vec{v}} = \ddot{\vec{x}}$$

$$m\ddot{\vec{x}} = m\vec{g} + \underbrace{c'}_{mc}(\vec{w} - \dot{\vec{x}})\|\vec{w} - \dot{\vec{x}}\| \quad / : m$$

equation of motion of projectile $\longrightarrow$ $\ddot{\vec{x}} = \vec{g} + c(\vec{w} - \dot{\vec{x}})\|\vec{w} - \dot{\vec{x}}\|$

(b)
$$\left.\begin{aligned} \dot{\vec{x}} &= \vec{v} \\ \dot{\vec{v}} &= g + c(\vec{w} - \vec{v})\|\vec{w} - \vec{v}\| \end{aligned}\right\} \text{ a system of 6 DE of } 1^{st} \text{ order.}$$

(c) Since the cannon is stationary at (0, 0, 0), initial conditions are:

$$\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v}(0) = v_0 \begin{bmatrix} \cos\varphi \cos\vartheta \\ \sin\varphi \cos\vartheta \\ \sin\vartheta \end{bmatrix}$$

$$\dot{\vec{Y}} = \vec{F}(t, \vec{Y}) \quad \dots \quad \vec{Y} = \begin{bmatrix} \vec{x} \\ \vec{v} \end{bmatrix}, \quad \vec{F}(t, \vec{Y}) = \begin{bmatrix} \vec{v} \\ \vec{g} + c(\vec{w} - \vec{v})\|\vec{w} - \vec{v}\| \end{bmatrix}$$

How to choose φ, ϑ to precisely hit the target T(400, 300)?

Our solution to the system of DEs determines a function

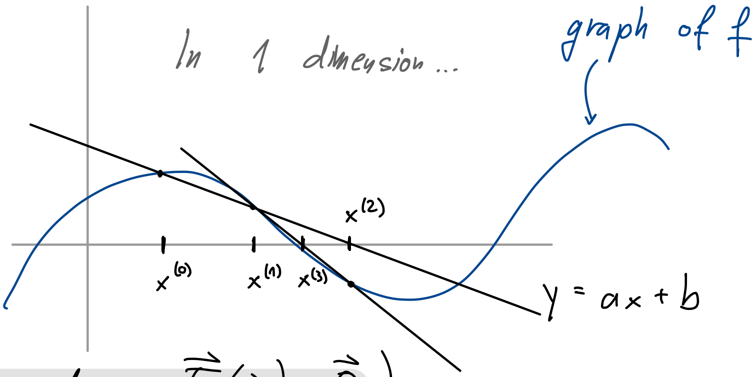
$$\begin{bmatrix} \varphi \\ \vartheta \end{bmatrix} \longmapsto \begin{bmatrix} x \\ y \end{bmatrix} = \vec{G}\left(\begin{bmatrix} \varphi \\ \vartheta \end{bmatrix}\right)$$

To hit the target, we need to solve $\vec{G}\left(\begin{bmatrix} \varphi \\ \vartheta \end{bmatrix}\right) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ ...

use the Newton's method $\longrightarrow$ ... $\vec{G}\left(\begin{bmatrix} \varphi \\ \vartheta \end{bmatrix}\right) - \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \vec{0}$.

(What would be $J\vec{G}$? ...)

Let's use the secant method:



In several dimension:

try to approximate

$\vec{F} : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ (try to solve $\vec{F}(\vec{x}) = \vec{0}$)

with an affine function $\vec{x} \longmapsto A\vec{x} + \vec{b}$. To do this, we need $n+1$ initial approximations, to get A and $\vec{b}$ from the system:

$$\left.\begin{aligned} A\vec{x}^{(0)} + \vec{b} &= \vec{F}(\vec{x}^{(0)}) \\ A\vec{x}^{(1)} + \vec{b} &= \vec{F}(\vec{x}^{(1)}) \\ &\vdots \\ A\vec{x}^{(n)} + \vec{b} &= \vec{F}(\vec{x}^{(n)}) \end{aligned}\right\} \quad n^2+n \text{ equations}$$

Once we solve this, solve $A\vec{x} + \vec{b} = \vec{0}$ ... $A\vec{x} = -\vec{b}$, the solution is $\vec{x}^{(n+1)}$ $\leftarrow$ which is a better approximation to the solution of $\vec{F}(\vec{x}) = \vec{0}$.