

# Mathematical Modelling Exam

30. 06. 2021

This is an open book exam. You are allowed to use your notes, books and any other literature. You are NOT allowed to use any communication device. You have 100 minutes to solve the problems.

1. We are given the matrix  $A$  and the vector  $b$ :

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}.$$

- (a) What is the rank of  $A$ ?  
(b) Compute one generalized inverse of  $A$ .  
(c) Determine all solutions of the system  $Ax = b$ .
2. Using one step of Newton's method approximate the solution of the system

$$\sin x + \cos y + e^{xy} = \arctan(x + y) - xy = 0,$$

with the initial approximation  $(x_0, y_0) = (0, 0)$

3. Let

$$r(t) = (2 \sin(2t), 2 \cos(2t), 3t)$$

be the curve in parametric coordinate with  $t \in [0, 2\pi]$ .

- (a) Sketch the curve in  $\mathbb{R}^3$ .  
(b) Sketch all three projections of the curve in the  $xy$ -,  $xz$ - and  $yz$ -coordinate planes.  
(c) Compute the arc length of the curve.

(The sketches do not need to be very precise.)

4. Solve the differential equation

$$y'' - 4y' + 5y = 8 \cos x.$$

Find a solution to this DE which has a local extremum in the point  $(0, 2)$ .