Assignment 2

Solve the following four exercises. Total: 15 points. Solutions must be submitted by 12.4.2026. Use the link on e-ucilnica to turn in your work. The submission must be in .pdf format.

Drop sort

What is the expected number of non zero elements in array out[] at the end of the function given below? Assume that array[] only has positive, non zero values. Solve using indicator random variables.

Express your final answer using the approximation $H_n = \ln n + 0.58$ for the harmonic number. (3 points)

Algorithm 1 Drop sort

1: **function** DROPSORT(int array[])
2:      ascending = true
3:      **if** array[0] > array[1] **then**
4:          ascending = false
5:      **end if**
6:      int[] out = new int[array.length] //Initializes all values to 0
7:      out[0] = array[0]
8:      out[1] = array[1]
9:      int c = 1
10:     **for** i = 2; i < array.length(); i++ **do**
11:         **if** ascending **then**
12:             **if** array[i] ≥ out[c] **then**
13:                 c++
14:                 out[c] = array[i]
15:             **end if**
16:         **else**
17:             **if** array[i] < out[c] **then**
18:                 c++
19:                 out[c] = array[i]
20:             **end if**
21:         **end if**
22:     **end for**
23:     **return** out
24: **end function**

Bloom filter

A Bloom filter is a data structure for checking if an element is a member of a set. It has two functions *add(element)* and *check(element)*. Function *add(element)* remembers each added element. Function *check(element)* returns $TRUE$ if the element is in the data structure and $FALSE$ otherwise. If the *check* function returns $FALSE$ the element is not in the Bloom filter. If it returns $TRUE$ there is a small probability that the element is not in the structure, meaning the algorithm can return a false positive.

A Bloom filter consists of an array of size n bits initialized to 0 and k hash functions. Each time an element is added, each hash function maps the element to one of n bits and sets that bit to 1. To check if an element was added the same hash functions are used to find the mapping of the bits. If any of the bits is 0, the Bloom filter returns $FALSE$, otherwise $TRUE$.

Assume $k = 5$ and that we insert n elements into a Bloom filter of size n. Also, assume that we are using perfect independent hash functions.

a) How many bits are set on 1 based on the above description? (1 point)

b) What is the probability of a false positive? (2 points)

Sticker album

You own a sticker album that has n stickers. How many stickers do you need on average to fill the album without exchanging stickers with other sticker collectors? Assume that the probability of obtaining each sticker is equal for each sticker. Solve using indicator variables. (4 points)

Annihilator method

a) Solve the following recurrence using the annihilator method. Find the exact solution. (2 points)

$$T(n) = T(n-1) + n^2 + n - 1, \qquad T(0) = 1 \tag{1}$$

b) Solve the following recurrence using the annihilator method. Find the exact solution. (3 points)

$$T(n) = 9T(n-1) - 28T(n-2) + 36T(n-3) - 16T(n-4)$$
$$T(0) = 2, \qquad T(1) = 8, \qquad T(2) = 28, \qquad T(3) = 112$$