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Collective fish behaviour

A Hydrodynamic Interaction Model

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Collective behaviour course research seminar report

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In this report we focused on modeling collective fish behaviour, taking into account hydrodynamics. First we took a look at what others have done in this field. Then we presented the mathematical model that models the fish behaviour and also explained how hydrodynamics come into play. Then we implemented the algorithm and presented the four stages of collective fish behaviour that can be seen from this - swarming, schooling, milling and turning, with the last one arising only when taking into account hydrodynamics.

Collective Fish Behavior | Hydrodynamic Interaction Modeling | Computational Simulation |Self-Propelled Particle Models | Behavioral Patterns in Schooling Fish

1. Introduction

Collective behaviour is a branch of computer science that attempts to model, recreate and visualize the behaviours of groups of animals, such as birds, sheep and insects. In this paper we will be focusing on the collective behaviour of fish, called *schooling*. It has been modeled numerous times before, but very rarely do we see that water physics are also taken into account. We will present one way of modelling the effect of hydrodynamics on the schooling of fish. We will also explain all the techniques used and also present what we plan to do in the future to test how this affects fish behaviour and further improve our model. Our research will be based on [1]. We plan to implement everything presented in this paper and further expand it.

2. Methods

In this section we first address previous work in fish behavioural modelling by conducting a literature review. We then explain the theoretical background from our paper of choice.

Literature review. Early studies of swarm behaviour, such as fish behaviour, employed mathematical models to simulate and understand the behaviour. The simplest mathematical models of animal swarms generally represent individual animals as individuals following rules of similar directional movement, proximity and collision avoidance with their neighbours. One such early example is the boids computer program created by Craig Reynolds [2], which simulates swarm behaviour following the above rules.

Within the specific domain of fish behavior, some of the early models expanded on the general swarm behaviour and introduced further complexities. For instance, some models focused on calculating velocity and angle based on probability distributions of random influences, as presented in a notable paper [3].

As research progressed, researchers undertook the challenge of modeling specific fish behavioral factors, including schooling, swarming, and milling. A particular study [4] analyzed all conceivable initial states to discern transitions between stationary states, such as schooling, swarming, or milling. A significant finding from this research highlighted that fish density in certain stationary states causes global interactions, where each fish perceives the presence of all others. The swarm algorithm proposed in this study adheres to the Lagrangian approach.

Recently, the research has expanded and includes various machine learning techniques to further improve the understanding and modelling of fish swarm behaviour. A case in point is a recent paper [5], wherein machine learning and computer vision methodologies were employed to track and gather fish pattern data for constructing a fish movement model.

Apart from new emerging modelling techniques, traditional mathematical models often overlook or oversimplify the intricate hydrodynamic interactions among fish. This is where we hope to improve and expand the research by implementing and improving the fish model to support hydrodynamic interactions, as proposed by [1].

Fish behaviour model. To model fish we will use so-called *self-propelled particle (SPP) models*, which can be constructed from simple rules to induce relatively complex behaviour. Each fish will be modeled as a particle moving around in a plane. It will move forward at some constant velocity v. Now we want to introduce interaction between a fish and its neighbours. All of the spacial parameters that we will use to achieve this are shown in the figure 2 and we will use them in the explanation later on.

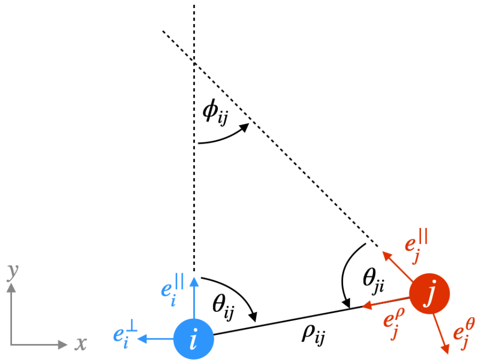

Figure 1. Visualization of the parameters used to model the interaction between fish i and j.

Firstly, we will add an attraction factor $k_p[m^{-1}s^{-1}]$ that attracts a fish towards its nearest neighbours and an alignment factor $k_v[m^{-1}]$ that makes a fish align with its neighbours. We will also add some Gaussian-distributed rotational noise σ to introduce randomness. This is approximately how fish schools are usually modeled. On top of this, we will add a fish's response to flow disturbance from other fishes. This is represented by an elementary dipole (a flow of a shape that is similar to the shape of a magnet's force field) with intensity Sv where $S = \pi r_0^2$ is the surface of a fish with length r_0. We have previously stated that fish will be modeled as points, but in terms of hydrodynamics we need to model them as objects with a surface. Now we will also introduce some new variables for readability purposes: $I_{||} = k_v\sqrt{\frac{v}{k_p}}$ represents alignment, $I_n = \sigma(vk_p)^{-\frac{1}{4}}$ represents noise and $I_f = S\frac{k_p}{v}$ represents dipole intensity. We can put all of this together to obtain motion equations:

$$\dot{r}_i = e_i^{||} + U_i \tag{1}$$
$$\dot{\theta}_i = \langle \rho_{ij}\sin(\theta_{ij}) + I_{||}\sin(\phi_{ij})\rangle + I_n\eta + \Omega_i \tag{2}$$

The equation 1 represents the movement of a fish from $\dot{r}_i$ at constant speed in the direction of its orientation $e_i^{||}$. We will call U_i the *drift term* that takes into account hydrodynamics. It is defined as:

$$U_i = \sum_{j\neq i} = u_{ji}, \quad u_{ji} = \frac{I_f}{\pi}\frac{e_j^{\theta}\sin(\theta_{ji}) + e_j^{\rho}\cos(\theta_{ji})}{\rho_{ij}^2}.$$

Each fish generates a flow field and u_{ji} is the field velocity generated by the j-th fish, affecting i-th fish. The spacial relation between a pair of fish is represented with polar coordinates in the framework of the j-th fish, hence the angles in the expression.

The equation 2 represents the rotation of a fish. The term η represents a *standard Wiener process* (a stochastic process used to model noise and disturbances) that is multiplied by the noise term. This introduces a model of free will in a fish. Ω_i is the rotation introduced by hydrodynamics and is defined as

$$\Omega_i = \sum_{j\neq i} e_i^{||} \cdot \Delta u_{ji} \cdot e_i^{\perp}.$$

This essentially means taking the gradient of $u_j i$ along x and y axis and multiplying it with the directions of a fish.

The notation $\langle\star\rangle$ indicates the averaging of all terms over the *Voronoi neighbours* (ν_i) (a selection of neighbours based on Voronoi diagrams) of a fish, weighted with $1 + \cos(\phi_{ij})$:

$$\langle\star\rangle = \frac{\sum\limits_{j\in\nu_i} \star(1+\cos(\theta ij))}{\sum\limits_{j\in\nu_i}(1+\cos(\theta ij))}$$

Figure 2. Visualization of the four distinct phases in swimming fish (upper left - swarming, upper right - schooling, lower left - milling, lower right - turning). Each image also represents the used parameters.

Implementation. After mutual consideration, Python was chosen as the primary programming language used for the implementation of the model. Its versatility and the extensive library support made it the ideal choice for our simulation's needs.

Key Libraries Used:

- *NumPy* for efficient array and numerical operations.
- *SciPy* for finding Voronoi neighbors
- *DearPyGUI* for a modern, easy-to-use, and fast GUI framework for Python.

Implementation Highlights:

1. *Modular Design:* We structured the simulation into distinct modules, each handling specific aspects like data input, processing, and output visualization. This approach improved maintainability and scalability.

2. *Optimization techniques:* To enhance performance, we employed various efficient data structures and algorithms, particularly in the processing of simulation data. We used *NumPy*'s built-in parallelism functionalities to make the simulation run even faster.

3. *Interactive User Interface:* A simple and functional GUI was developed using DearPyGUI, enabling users to interact with the simulation and adjust parameters in real-time.

3. Results

Our implementation of the described fish behavior model has enabled us to successfully replicate key stages of fish swimming, as outlined in the scientific paper [1]. The model accurately captures the intricate dynamics of milling, swarming, schooling, and turning behaviors, through the setting of different parameters

Swarming, a behavior characterized by the formation of sparse groups without a discernible orientation (figure 2 - top left), emerges prominently when the noise level is comparable to or exceeds the alignment factor ($k_p \leq \sigma$). This setting of the parameters results in a dynamic simulation where fish exhibit cohesive yet uncoordinated movements, akin to their real-world counterparts during non-directed motion.

Further, the model successfully emulates *schooling* behavior, as depicted in figure 2 (top right), which leads to the creation of denser fish groups moving in a specific direction. This behavior is depicted when the alignment factor dominates over noise ($k_p > \sigma$). Through fine-tuning parameters, the simulation portrays cohesive and directional movement similar to natural schooling behavior observed in fish populations.

Milling, represented in figure 2 (bottom left), manifests as a vortex-like pattern in fish movements. This behavior arises when alignment and attraction factors become comparable ($k_p \sim k_v$), while maintaining a relatively low noise level (σ). The model

	$P \leq 0.5$	$P > 0.5$
$M \leq 0.4$	swarming	schooling
$M > 0.4$	milling	turning

Table 1. Classification of fish behaviour phases

aptly reproduces this phenomenon, portraying dynamic swirling motions reminiscent of milling behaviors observed in certain fish species.

Finally looking at the *turning* behaviour in figure 2 (bottom right), which can only be observed with the incorporation of hydrodynamics into the model, we can see fish groups following a larger circular trajectory. In order to see this behaviour, the model parameters must reach a specific value. This behaviour usually arises when transitioning between different behaviours, for example right before when fish enter milling. The model's ability to replicate this behavior highlights its complexity, showcasing how neighbouring fish dynamics and specific parameter settings influence the collective movement of fish.

Additionally, we classified the four stages using the parameters P (polarization) and M (milling) defined as:

$$P = |\overline{e_i^{||}}| \quad M = \frac{|\overline{e_i^r \times \dot{r_i}}|}{|\overline{e_i^r}||\dot{r_i}|},$$

where e_i^r is the vector from the center of mass to the i-th fish and r_i is the position of the fish. The classification can be seen in table 1, where thresholds were provided in the article [1].

In addition to successfully implementing the fish behavior model based on the paper [1], we've integrated a user-friendly graphical interface where users can change parameters and observe fish dynamics in real time. To enhance realism, we've also implemented collision checking to prevent fish overlap and introduced bounding box logic for custom interactions with simulation boundaries, which can be set to either repulsion, blocking or making fish loop around.

Key accomplishments at this stage include:

- Full implementation of the model based on the paper [1].
- Implementation of graphical user interface for real time parameter changes and observations.
- Replication of many results from [1]

4. Discussion

We have successfully implemented a simulation of fish behaviour based on a mathematical attraction-alignment model taking into account hydrodynamics. We are pleased with the result, as we were able to replicate the behaviours presented in the article [1] and make it run in real time as an interactive simulation. During the development we struggled quite a bit with the implementation of the model, as some things were not that clearly explained in the article, for example the rotation induced by hydrodynamics (Ω_i). A big problem is that we could not explicitly determine if our implementation was correct and we could only rely on recognising patterns of fish behaviour visually. But all in all, the implementation was successful at the end. In the future we strive to improve the model and add our own features, like external water flows and a predator. We would also like to implement some metrics to see if fish swim better and faster when hydrodynamics are taken into account.

CONTRIBUTIONS. GK wrote the introduction, described the collective behaviour model and wrote the discussion. AČ wrote abstract and results. JP wrote the literature review. MŠ reviewed the report. AČ and JP created basic foundation for the implementation as a proof of concept. MŠ and GK implemented the behaviour of fish, added the graphical interface and polished the implementation.

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