Computational topology
Lab work 14

1. Let $A = (1,3)$, $B = (2,4)$, $C = (1,2)$ $D = (2,5)$ and $E = (1,\infty)$. For each of the pairs X_i, Y_i of persistent diagrams given below

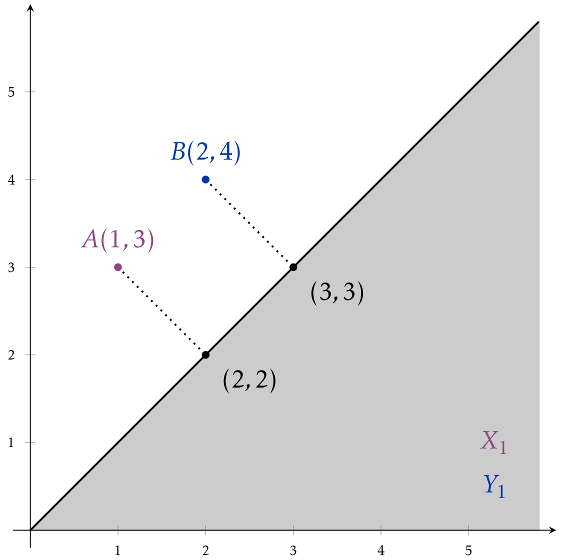
   - find all bijections $\eta\colon X_i \to Y_i$,
   - determine $\|x - \eta(x)\|_\infty$ for each bijection and for all $x \in X_i$ and
   - calculate the bottleneck distances $W_\infty(X_i, Y_i)$ and Wasserstein distances $W_q(X_i, Y_i)$ for $q = 1, 2$.

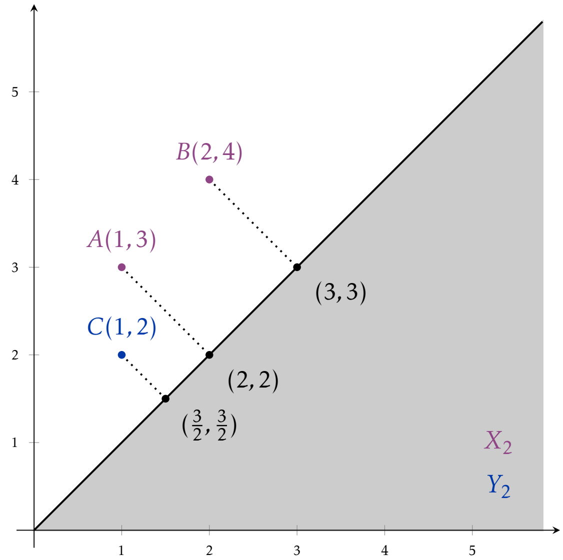
   (a) $X_1 = \Delta \cup \{A\}$, $Y_1 = \Delta \cup \{B\}$,

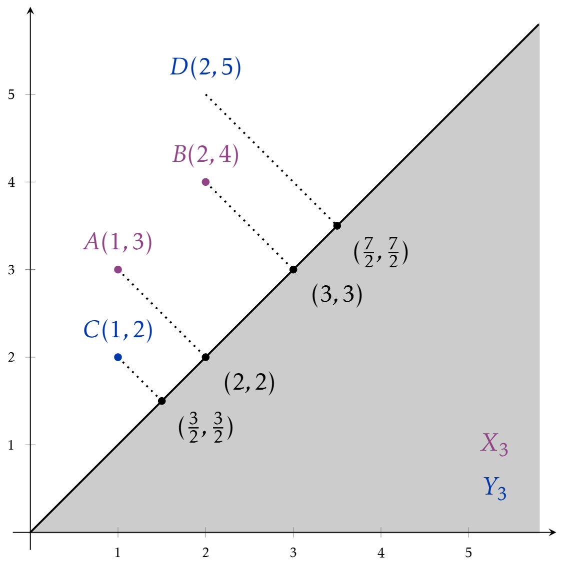
   (b) $X_2 = \Delta \cup \{A, B\}$, $Y_2 = \Delta \cup \{C\}$,

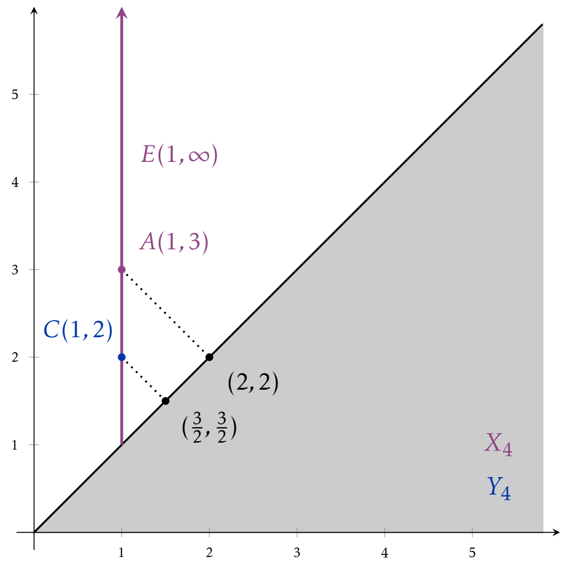
   (c) $X_3 = \Delta \cup \{A, B\}$, $Y_3 = \Delta \cup \{C, D\}$,

   (d) $X_4 = \Delta \cup \{A, E\}$, $Y_4 = \Delta \cup \{C\}$.









The **bottleneck distance** between persistence diagrams X and Y:

$$W_\infty(X,Y) = \inf_{\eta\colon X\to Y}\left(\sup_{x\in X}\|x-\eta(x)\|_\infty\right).$$

The **Wasserstein distance** for all $q \in \mathbb{R}$:

$$W_q(X,Y) = \left(\inf_{\eta\colon X\to Y}\sum_{x\in X}\|x-\eta(x)\|_\infty^q\right)^{\frac{1}{q}}.$$