Mathematical modelling, Exam 3

22. 8. 2019

- 1. (a) Construct any non-diagonal 3×2 matrix A whose singular values are 2 and 1.
 - (b) Find the Moore-Penrose inverse A^+ .
 - (c) Let $b \in \mathbb{R}^2$. Describe the property uniquely characterizing point $A^+ \cdot b$ with respect to the system Ax = b.
- 2. Two surfaces in the upper halfspace z > 0 are given by the following equations:

$$\Pi: x^2 + y^2 = \frac{z^2}{2} \qquad \Sigma: x^2 + y^2 = z.$$

Curve γ is the intersection of surfaces Π and Σ . Let $P = (1, 1, 2) \in \gamma$.

- (a) Find the angle at which the surfaces intersect at P.
- (b) Find the line tangent to γ at P.
- (c) Find the plane that is tangent to Σ at (1, 2, 5).
- 3. Solve the following exact differential equation $2xy + (x^2 + 3y^2)y' = 0$.
- 4. Solve the differential equation $y'' + 9y = 2x^2 1$. with the initial condition y(0) = y'(0) = 1.