## Mathematical modelling, Exam 3

## 22. 8. 2019

1. (a) Construct any non-diagonal $3 \times 2$ matrix $A$ whose singular values are 2 and 1 .
(b) Find the Moore-Penrose inverse $A^{+}$.
(c) Let $b \in \mathbb{R}^{2}$. Describe the property uniquely characterizing point $A^{+} \cdot b$ with respect to the system $A x=b$.
2. Two surfaces in the upper halfspace $z>0$ are given by the following equations:

$$
\Pi: x^{2}+y^{2}=\frac{z^{2}}{2} \quad \Sigma: x^{2}+y^{2}=z .
$$

Curve $\gamma$ is the intersection of surfaces $\Pi$ and $\Sigma$. Let $P=(1,1,2) \in \gamma$.
(a) Find the angle at which the surfaces intersect at $P$.
(b) Find the line tangent to $\gamma$ at $P$.
(c) Find the plane that is tangent to $\Sigma$ at $(1,2,5)$.
3. Solve the following exact differential equation $2 x y+\left(x^{2}+3 y^{2}\right) y^{\prime}=0$.
4. Solve the differential equation $y^{\prime \prime}+9 y=2 x^{2}-1$. with the initial condition $y(0)=y^{\prime}(0)=1$.

