Name and surname: _____ Student ID: _____

	1	2a	2b	3a	3b	3c	3d	4a	4b	Sum
Points										

EXAM in Mathematical Modelling
June 13, 2025

For each of tasks justify all your answers.

1. **[12 points]** You are helping a landlord estimate monthly rent (unit: €100) based on the location (ratings 1, 2 or 3) room size (in $20m^2$) using past data from 3 rental units:

Room	**Location** (1-3)	**Size** (unit: $20m^2$)	**Rent** (€100)
A	1	1	3
B	1	2	5
C	2	1	6

You will use a model in which rent is a linear function in location rating and the room size:

$$\text{Rent} = a \cdot \text{Location} + b \cdot \text{Size}.$$

Find the function of this form that fits the data best according to the least squares criterion and advise the price for the rent of a room of $40m^2$ on location of rating 3.

2. Let $\mathbf{a} = (1, 0, \pi)$ and consider the function $\vec{F}\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$\vec{F}(x, y, z) = \begin{bmatrix} x \\ yz \\ \sin(xz) \end{bmatrix}.$$

a. **[2 points]**Compute the Jacobian matrix of $\vec{F}$ at the point $\mathbf{a} = (1, 0, \pi)$.

b. **[10 points]** Consider the system

$$\begin{aligned} x &= 1 \\ yz &= 2 \\ \sin(xz) &= 1 \end{aligned}$$

Do one step of Newton's method to find an approximate solution to this system starting with initial guess $(x_0, y_0, z_0) = \mathbf{a}$.

3. Consider the parametrized curve $\mathcal{C} \subset \mathbb{R}^2$ given by:

$$x(t) = 2\cos(t), \quad y(t) = \sin^2(t)$$

a. **[3 points]**Identify all singular points on the curve $\mathcal{C}$.

b. **[3 points]** Determine the points on $\mathcal{C}$ where the tangent line is parallel to either the x-axis or the y-axis.

c. **[4 points]**Sketch the curve $\mathcal{C}$ in the plane.

d. **[4 points]** Compute the area of the figure in the first quadrant enclosed by the curve $\mathcal{C}$, the x-axis, and the y-axis.

4. On a peaceful island of Lushlandia, a population of rabbits is thriving. The island's climate and resources are perfect for steady growth: the rabbit population increases naturally at a rate proportional to its current size. The natural growth rate of the population is 5% per year.

   But there's more! A nearby island, Hopperia, sends a constant number of 1000 rabbits per year to Lushlandia as part of a wildlife relocation program.

   a. **[4 points]** Write an equation to model the population of rabbits $R(t)$ on Lushlandia as a function of time, which is measured in years.

   b. **[8 points]** Exactly 20 years ago Lushlandia celebrated the population of 10000 rabbits. Approximately how many are there this year?