

Computational Topology

Exam 2, July 3, 2019

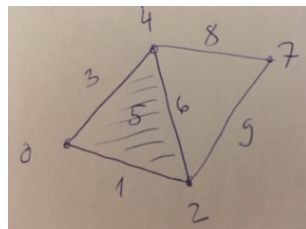
You have approx. 90 minutes to solve the problems.

This is an open book exam. You are allowed to use books and notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. The following sets are a cover of $\{p, q, r, x, y, z, w\}$: $A = \{r, x, z\}$, $B = \{q, w, z\}$, $C = \{p, q, y, z\}$, $D = \{x, y\}$, $E = \{p, w\}$. Let simplicial complex K be the nerve of this cover.
 - Draw K .
 - Write down the star $\text{St}(A)$ and the link $\text{Lk}(A)$ of the vertex A in K .
 - Write down the matrices for the boundary operators $\partial_2 : C_2 \rightarrow C_1$ and $\partial_1 : C_1 \rightarrow C_0$.
 - Write down a set of generators for the cycle group Z_1 .
 - What are the Betti numbers of K ?
2. Let X be a finite set.
 - (a) Write down the definition of an abstract simplicial complex on the set X .
 - (b) Let K be an abstract simplicial complex on X and suppose A, B are subcomplexes of K . Prove that $A \cap B$ is an abstract simplicial complex, but $A \setminus B$ is not.
3. Explain the difference between the concepts of homotopy equivalence and homeomorphism. Let C be a circle. Draw a simplicial complex K such that
 - $|K|$ is homeomorphic to C ,
 - $|K|$ is homotopy equivalent but not homeomorphic to C ,
 - $|K|$ is not homotopy equivalent to C .

How would you prove that your examples are good? Describe a simple (preferably but not necessarily topological) test in each case.

4. The picture shows a simplicial complex K with values of a discrete Morse function on the simplices:



- Draw the corresponding discrete vector field.
- What is the sublevel complex K_a at $a = 5$?
- Draw the barcodes in dimension 0 and 1 for the sublevel complex filtration.