Computational Topology

Exam 1, June 9, 2014

You have 60 minutes to solve the problems.

This is an open book exam. You are allowed to use books and notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. Let A and B be subsets of the plane $\mathbb{R}^2$. Write down the definition of a

   - homeomorphism $f\ A \to B$
   - homotopy equivalence $f\ A \to B$

   Classify the digits 0,1,2,3,4,5,6,7,8,9 (as they are written here) up to

   - homeomorphism
   - homotopy equivalence

2. Given the sample of points $S = \{x_1 = (0,0), x_2 = (1,1), x_3 = (2,3), x_4 = (-1,2), x_5 = (3,-1), x_6 = (4,2)\}$

   (a) write down the first edge and the first triangle which appear in the line sweep triangulation algorithm if the sweep is from left to right,

   (b) Is the edge $< x_2, x_3 >$ an edge of the Dealaunay triangulation on S? What about the edge $< x_4, x_6 >$? Why?

3. Given the set S from the previous problem, list all the simplices of maximal dimension of the following simplicial complexes on S. In ech case give the dimension of the simplicial complex.

   (a) Vietoris Rips complex with $r = 1$,

   (b) Vietoris Rips complex with $r = 3$,

   (c) Čech complex with $r = 1$,

   (d) Čech complex with $r = 3$,

   (e) Čech complex with $r = 10$.

4. How is the boundary homomorphism in the Morse chain complex defined? Given the following simplicial complex:

   (a) find a Morse pairing on the simplices,

   (b) write down the corresponding Morse chain complex and determine the boundary homomophisms.

   (c) What are its Betti numbers?

   (d) Is your Morse pairing perfect?

5. Write down the barcode and the persistent diagram for the upward sweep of the digit 6 (as it is written here).