Computational Topology

Exam 2, June 24, 2014

You have 60 minutes to solve the problems.

This is an open book exam. You are allowed to use books and notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. Let f and g be two maps from A to B.
   (a) What is a homotopy from f to g?
   (b) Let $A = \{(x, 0), 0 \leq x \leq 1\} \subset \mathbb{R}^2$ and let $B = \mathbb{R}^2$.
      i. Does there exist a homotopy between the constant map $f(x, 0) = (0, 0)$ and the map $g(x, 0) = (x, x)$? Can you write down such a homotopy, if it exists?
      ii. What about a homotopy between the constant map $f(x, 0) = (0, 0)$ and the map $g(x, 0) = e^{i2\pi x}$?
2. Given the sample of points $S = \{x_1 = (0, 0), x_2 = (1, 1), x_3 = (2, 3), x_4 = (-1, 2), x_5 = (3, -1), x_6 = (4, 2)\}$,
   (a) Which Voronoi cell or cells does the point $(3, 3)$ belong to?
   (b) Does the edge $< x_3, x_4 >$ belong to the Delaunay triangulation on S?
   (c) What about the triangle $< x_1, x_4, x_2 >$?
   (d) Which simplices of the Delaunay triangulation on S are in the star of the vertex x_1?
3. Given the simplicial complex

$$K = \{v0, v1, v2, v3, v4, \langle v0, v1\rangle, \langle v0, v2\rangle, \langle v0, v3\rangle, \langle v1, v2\rangle, \langle v1, v3\rangle, \langle v0, v1, v2\rangle\}$$

   define a function f on K which assignes the following values to the simplices of K (in the above order):

$$\{0, 2, 2, 0, 4, 1, 5, 4, 3, 1, 4\}.$$

   (a) Is f a discrete Morse function? Why?
   (b) If it is, draw the corresponding discrete vector field and list the critical simplices.
   (c) Write down the Morse chain complex and the boundary operator in dimension 1, ∂_1.
   (d) Write down the Betti numbers of K.
4. Write down the filtration by sublevel complexes for the function f on the simplicial complex from the previous problem and draw the barcodes in dimension 0 and 1.