Computational Topology

Exam 1, June 13, 2019

You have approx. 90 minutes to solve the problems.

This is an open book exam. You are allowed to use books and notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. Let $X = \{(0,0), (0,.5), (0,1), (1,0), (1,1), (2,0)\} \subset \mathbb{R}^2$.
   - Draw $K = \mathrm{Rips}(X, 1.1)$.
   - Write down the star $\mathrm{St}(A)$ and the link $\mathrm{Lk}(A)$ of the vertex $A = (0,0)$ in K.
   - Write down the matrices for the boundary operators $\partial_2 : C_2 \to C_1$ and $\partial_1 : C_1 \to C_0$
   - Write down a set of generators for the cycle group Z_1.
   - What are the Betti numbers of K?
2. (a) Draw a triangulation and compute the Euler characteristic of the Moebius band M.

   (b) The boundary of M consists of one curve, homeomorphic to a circle. Glue a disc D to M by identifying the boundary of D and M. The resulting space is called X. Find the Euler characteristic of X.

   (c) Write down the Betti numbers β_0, β_1 and β_2 of M and of X, and justify your answer.
3. Let $X \subset \mathbb{R}^2$ be a finite subset. Explain the difference between the Rips and the Cech complex of X at $r > 0$. Draw an example demonstrating the difference.