Linear Programming: Exercises

1. The Holiday Meal Turkey Ranch is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys. Each brand of feed contains, in varying proportions, some or all of the three nutritional ingredients essential for fattening turkeys. Each kilogram of brand 1 contains 5 grams of ingredient A, 4 grams of ingredient B and 0,5 grams of ingredient C. Each kilogram of brand 2 contains 10 grams of ingredient A, 3 grams of ingredient B, but nothing of ingredient C. The brand 1 feed costs the Ranch 20c a kilogram, while the brand 2 feed costs 30c a kilogram.

   The minimum monthly requirement per turkey is: 90 grams of ingredient A; 48 grams of ingredient B and 1,5 grams of ingredient C.

   Formulate an LP model to help the rancher decide how to mix the two brands of turkey feed so that the minimum monthly intake requirement for each nutritional ingredient is met at minimum cost.

   Use the graphical approach to solve this model.

2. Giapetto's Woodcarving, Inc., manufactures two types of wooden toys; soldiers and trains. A soldier sells for R27 and uses R10 worth of raw materials. Each soldier that is manufactured increases Giapetto's labour and overhead costs by R14. A train sells for R21 and uses R9 worth of raw materials. Each train built increases Giapetto's labour and overhead costs by R10. A train requires one hour of carpentry and one hour of finishing. A soldier requires two hours of finishing and one hour of carpentry. Only 100 finishing hours and 80 carpentry hours are available each week. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wishes to maximise weekly profits.

   Use the graphical approach to find the optimal solution.

3. The ACE Manufacturing Company produces two lines of its product, the "Super" and the "Regular". Resource requirements for production are as follows:

Product line	**Profit** (R/unit)	**Assembly time** (hours)	**Paint time** (hours)	**Inspection time** (hours)
Regular	50	1,0	0,4	0,2
Super	75	0,8	0,5	0,4

   There are 160 hours of assembly time and 80 hours of paint time available per week. The inspection on each product is done by a team of quality controllers who work 40 hours per week. From past data ACE's management knows that the demand for their products always exceeds the production, but they also know that they usually sell at least double as many "Regular" as "Super" products.

   Formulate as an LP model and use the graphical approach to solve it.

4. Solve the following LP models graphically. For each solution state clearly whether it is infeasibile, multiple, unbounded or degenerate and which constraints are binding, non-binding or redundant.

a) Maximise $C = 3A + 5B$

subject to

$$\begin{aligned} A \quad\quad\quad &\geq 5 & (1) \\ B &\leq 10 & (2) \\ A + 2B &\geq 10 & (3) \\ B &\geq 0. & \end{aligned}$$

b) Maximise $Z = P + Q$

subject to

$$\begin{aligned} P + 2Q &\leq 6 & (1) \\ 2P + Q &\leq 8 & (2) \\ P \quad\quad\quad &\geq 7 & (3) \\ Q &\geq 0. & \end{aligned}$$

c) Maximise $PROFIT = M + 2N$

subject to

$$\begin{aligned} M + N &\leq 25 & (1) \\ 2M + N &\leq 30 & (2) \\ N &\leq 35 & (3) \\ M, N &\geq 0. & \end{aligned}$$

d) Maximise $PROFIT = 5X + 2Y$

subject to

$$\begin{aligned} 7{,}5X + Y &\leq 15 & (1) \\ 5X + 2Y &\leq 20 & (2) \\ X, Y &\geq 0. & \end{aligned}$$

Solutions to Exercises

1. Holiday Meal Turkey Ranch data:

Ingredient	Composition of each kg of feed (grams)		Minimum monthly requirement (grams)
	Brand 1	Brand 2	
A	5	10	90
B	4	3	48
C	0,5	0	1,5
Cost per kg	20c	30c	

Let ONE = number of kg of brand 1 feed purchased monthly, and

TWO = number of kg of brand 2 feed purchased monthly.

Minimise $COST = 20ONE + 30TWO$

subject to

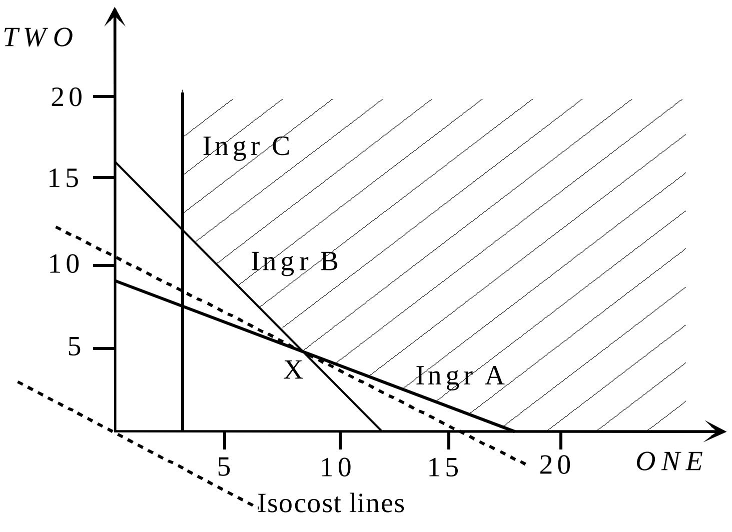
$5ONE + 10TWO \geq 90$ (Ingredient A)

$4ONE + 3TWO \geq 48$ (Ingredient B)

$0{,}5ONE \geq 1{,}5$ (Ingredient C)

$ONE, TWO \geq 0.$

Graphical solution:



The isocost lines enter the feasible area at X, i.e. the point (8,4; 4,8). To find the exact coordinates of this point, solve the equations representing the constraints intercepting in this point simultaneously.

The optimal solution:

The rancher must buy 8,4 kg of the brand 1 feed and 4,8 kg of the brand 2 feed monthly for a minimum cost of R3,12 per turkey.

2. Let *SOLDIER* = number of soldiers produced each week, and
TRAIN = number of trains produced each week

Total weekly revenue = revenue from soldiers + revenue from trains

$= 27SOLDIER + 21TRAIN.$

Weekly raw material costs $= 10SOLDIER + 9TRAIN.$

Other weekly variable costs $= 14SOLDIER + 10TRAIN.$

Total weekly costs $= 24SOLDIER + 19TRAIN.$

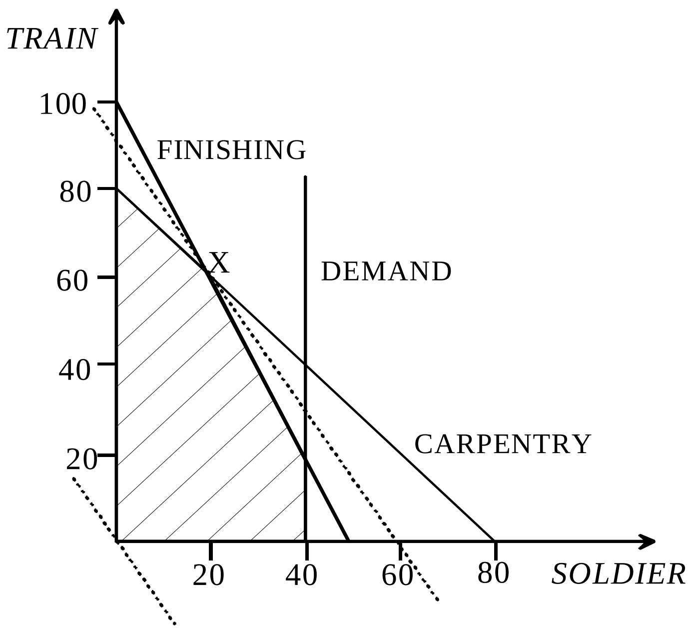
Giapetto wishes to maximise profit, with

$$PROFIT = (27SOLDIER + 21TRAIN) - (24SOLDIER + 19TRAIN)$$
$$= 3SOLDIER + 2TRAIN.$$

The LP model is:

Maximise $PROFIT = 3SOLDIER + 2TRAIN$
subject to

$2SOLDIER + TRAIN \leq 100$ (Finishing)
$SOLDIER + TRAIN \leq 80$ (Carpentry)
$SOLDIER \leq 40$ (Demand)
$SOLDIER, TRAIN \geq 0.$



The optimal solution is at point X, that is the point (20; 60).

The optimal solution is to produce 20 soldiers and 60 trains per week to make the maximum profit of R180.

3. Let SUP = number of units of the "Super" product to be manufactured, and

REG = number of units of the "Regular" product to be manufactured.

Maximise $PROFIT = 50REG + 75SUP$

subject to

$$\begin{aligned} REG + 0{,}8SUP &\le 160 && \text{(1. Assembly)} \\ 0{,}4REG + 0{,}5SUP &\le 80 && \text{(2. Paint)} \\ 0{,}2REG + 0{,}4SUP &\le 40 && \text{(3. Inspection)} \\ REG - 2{,}0SUP &\ge 0 && \text{(4. Demand)} \\ REG,\ SUP &\ge 0. \end{aligned}$$

Consider the demand constraint: If ACE knows that they usually sell at least double as many Regulars as Supers, it means that the selling ratio of Regular to Super is 2 to 1, that is

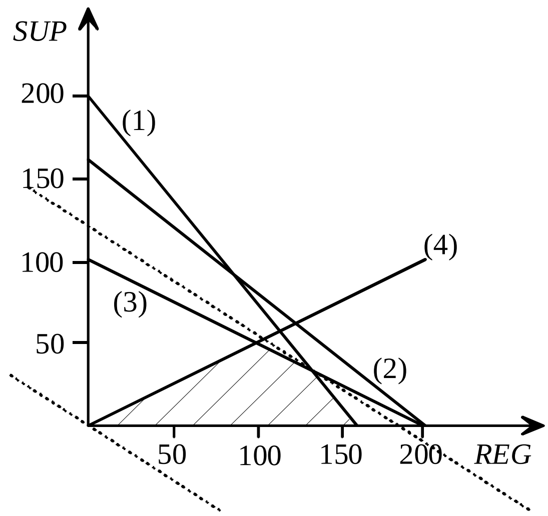
$$\frac{REG}{SUP} = \frac{2}{1} \quad \text{or} \quad REG = 2SUP\,.$$

And, because of the *at least*, it becomes $REG \ge 2SUP$.

The line representing this constraint passes through the point (0, 0). To determine another point, let REG take on any value, say 50, and find the value of SUP.

We now have two points (0, 0) and (50, 25) and we can draw the line.

To find the area satisfying the inequality, substitute any point, say (150; 50), into $REG \ge 2SUP$ which is $150 > 2 \times 50$. The inequality holds and we know that all the points on the same side as this point will satisfy the constraint.



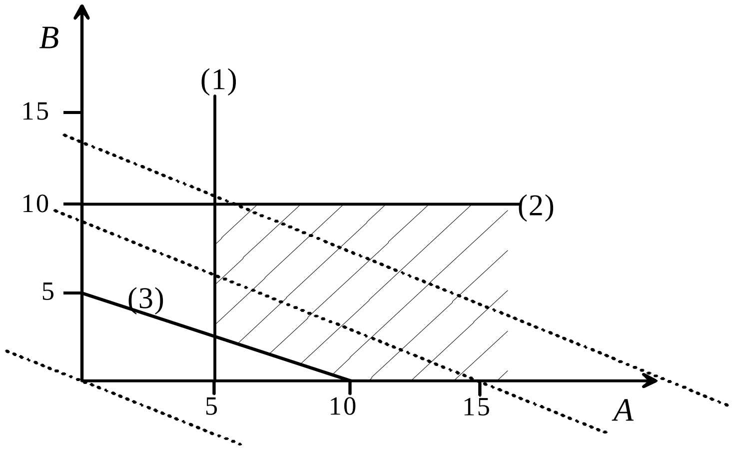
The optimal profit is found where the isoprofit lines leave the feasible area. This is the point where the inspection and assembly constraints intersect.

To find the exact value of this point, solve the simultaneous equations which represent constraints (1) and (3) to get $SUP = 33{,}3$ and $REG = 133{,}3$.

The optimal solution is to produce 33,3 units of the "Super" model and 133,3 units of the "Regular" model. The maximum profit is R9 162,50.

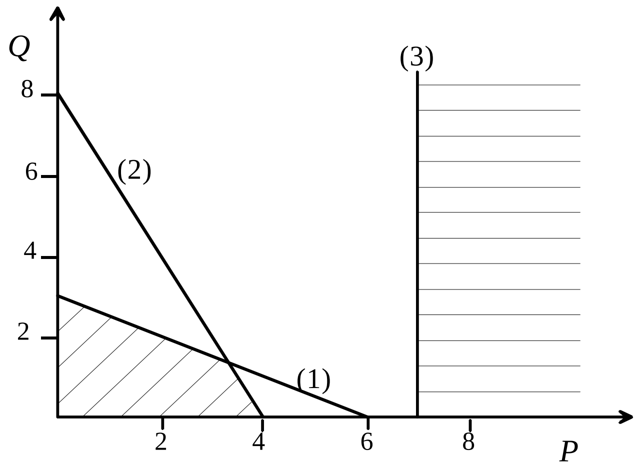
Constraint (2), the paint constraint, doesn't contribute to the feasible area – in fact, it doesn't touch it at all. This constraint is therefore redundant.

4.a) The graphical solution of the LP model:



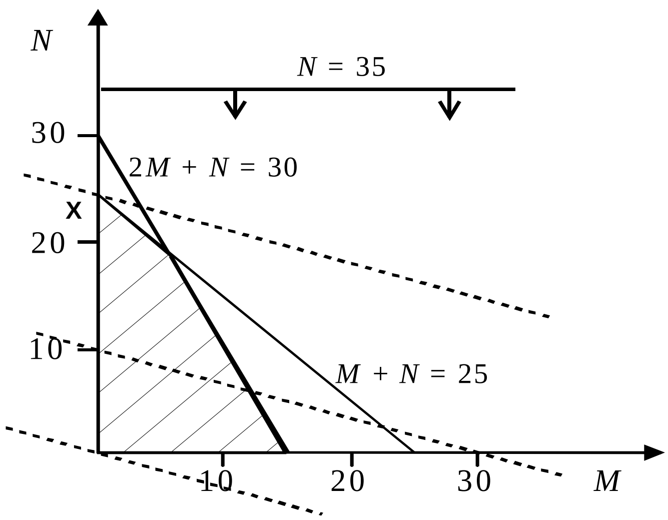
The solution area is unbounded to the right and the value of the objective function can increase indefinitely without ever reaching a maximum. The solution is unbounded.

b) The graphical presentation of the feasible area:



There is no area satisfying all the inequalities simultaneously and we say the LP model (or the problem) is infeasible. There is no solution to this LP model.

c) The graphical solution to the LP model:



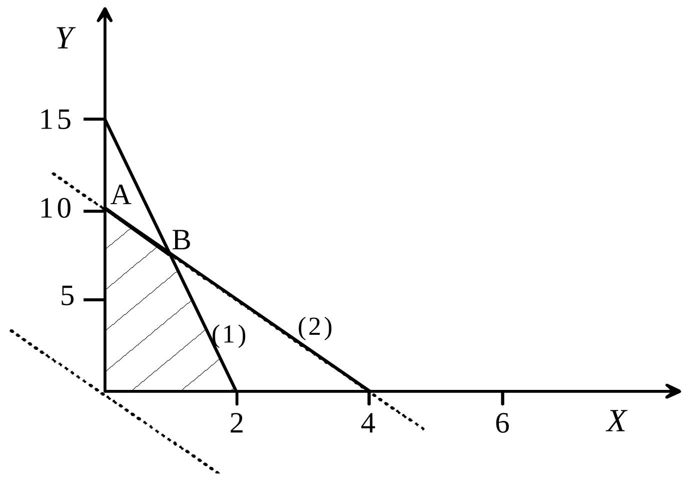
The optimal solution is at the point X. No units of M and 25 units of N should be produced for a maximum profit of 50 units.

Constraint $N \leq 35$ is redundant - it does not influence the feasible area at all.

Constraint $M + N \leq 25$ is binding - it passes through the optimal point. (No slack).

Constraint $2M + N \leq 30$ is non-binding - it does not go through the optimal point. (Slack of 5 units.)

d) The graphical solution:



The isoprofit lines are parallel to constraint (2). The optimal solution is therefore found on the line segment AB. Any point on the line segment AB gives the maximum profit of 20 units. The endpoints are (0, 10) and (1, 7,5).

This LP model has multiple optimal solutions.