· Name and surname: ________________________ Vpisna številka: ___________________

1. Exam for OMA, 23.01.2020

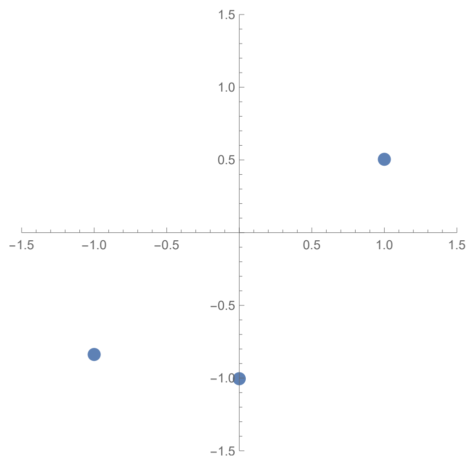
- Time for exam: **45 minut**
- If you do not have enough space to write an answer on the exam paper, mark this and continue on the additional paper.
- For passing grade you have to achieve at least 50% of all points and also 30% of the points at each task, i.e., 1.5 points out of 5 points. The number in the bracket [·] tells you how many points you get for a correct solution to the question.
- Any attempt of copying the solution of someone else, talking, using notes or electronic equipement is **strictly** forbidden.

1. [**5 points**] **Number sets**

(a) [**2**] Explain polar form of a complex number and write down the formula for exponentiation of a complex number in the polar form.

(b) i. [**2**] Given a complex equation $a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0 = 0$, where $a_n, \ldots, a_0 \in \mathbb{R}$ are real numbers, and $w \in \mathbb{C} \setminus \mathbb{R}$ one of its solutions. Find another solution of the equation and prove it is indeed a solution.

ii. [**1**] We are given an equation $z^6 - \frac{z^4}{18} - \frac{8z^3}{9} + \frac{17z^2}{16} - \frac{8z}{9} + \frac{305}{144} = 0$. The points on the sketch below are some of its solutions. Draw all the other solutions. (*Hint:* You do not have to solve the equation.)



2. **[5 points] Sequences and series**

(a) [**1**] Write down the definition of a limit of a sequence $\{a_n\}_{n\in\mathbb{N}}$, $a_n \in \mathbb{R}$.

(b) i. [**1**] Write down the so called sandwich theorem for limits of sequences.

ii. [**1**] Let $A = \sum_{n=0}^{\infty} a_n$ and $B = \sum_{n=0}^{\infty} b_n$ be series, where $a_n = \frac{2}{3^{n+1}}$ and $b_n = \frac{1}{2^n}$. Calculate the sums A and B?

iii. [**2**] Let $C = \sum_{n=0}^{\infty} c_n$ be a series where $c_n \in \{a_n, b_n\}$. Eg., $c_0 = b_0, c_1 = a_1, c_2 = b_2, c_3 = a_3, \ldots$ Bound the sum C from above and below with the use of A and B. Justify your answer.

3. **[5 točk] Functions**

(a) [**1**] Write down the definition of a left limit of a function $f : \mathbb{R} \to \mathbb{R}$ in the point $x_0 \in \mathbb{R}$.

(b) [**1**] Write down an example of a function $f : \mathbb{R} \to \mathbb{R}$ with a left and right limit in the point $x = 0$ which are not equal.

(c) [**3**] Find examples of functions $f, g : \mathbb{R} \to \mathbb{R}$ satisfying:

i. [**1**] g is not continuous in the point $a \in \mathbb{R}$, while $f \circ g$ is continuous in a.

ii. [**1**] $\lim_{x\to 0}(f \circ g)(x)$ exists, while $\lim_{x\to 0} f(x)$ does not.

iii. [**1**] $\lim_{x\to 0} g(x)$ exists, while $\lim_{x\to 0}(f\circ g)(x)$ does not.

4. [**5 točk**] **Differentiation**

(a) [**1**] Write down the definition of a level curve of a function $f:\mathbb{R}^2\to\mathbb{R}$.

(b) [**1**] Write down the definition of extreme values of the function $f:\mathbb{R}^2\to\mathbb{R}$ under the condition $g(x,y)=0$ where $g:\mathbb{R}^2\to\mathbb{R}$ is some function.

(c) [**3**] Let $f:\mathbb{R}^2\to\mathbb{R}$, $f(x,y)=x-3xy+3y^2$ be a function. For each condition $g(x,y)=0$ below determine whether the extreme values for f exist or not. Justify your answers, while you do not have to calculate the extrema.

i. [**1**] $g(x,y)=f(x,y)-10$.

ii. [**1**] $g(x,y)=y-x$.

iii. [**1**] $g(x,y)=x^2+y^2-1$.

5. [**5 točk**] **Integration**

(a) [**1**] Write down the definition of a definite integral of a function $f:[a,b]\to\mathbb{R}$.

(b) [**1**] Write down the definition of a generalized integral $\int_1^\infty f(x)dx$ of a function $f:[1,\infty)\to\mathbb{R}$ and give an example of a function for which it exists.

(c) **[1.5]** Does there exist a *continuous* function $f : [0,1] \to \mathbb{R}$, for which a definite integral $\int_0^1 f(x)dx$ exists but f does not have an indefinite integral on the interval $(0,1)$? If the answer is yes, then give an example justifying this or explain why the answer is no.

(d) **[1.5]** Does there exist a function $f : (0,1) \to \mathbb{R}$ which has an indefinite integral but for which the definite integral $\int_0^1 \widetilde{f}(x)dx$ does not exist for any extension $\widetilde{f} : [0,1] \to \mathbb{R}$ of a function f? If the answer is yes, then give an example justifying this or explain why the answer is no.

6. [**5 točk**] **Differential equations**

(a) **[1]** Write down the definition of orthogonal trajectories to the family of curves.

(b) **[4]** Find orthogonal trajectories to the family of parabolas $\dfrac{y^2}{x} = a$ where $a \in \mathbb{R}$ is a real parameter.