Osnove matematične analize: predrok

8. januar 2021

Write each problem onto a separate sheet of paper. If you are writing onto a blank piece of paper rather than the problem sheet, please sign your name at the top of every page, write the problem number at the top as well and scan the problems in the correct order. Thanks!

Student ID

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Question 1 (30 marks)

Let the sequence (a_n) be given by the recursive formula

$$a_{n+1} = \frac{a_n - 1}{a_n + 3},$$

and the initial term $a_1 = 2$.

a) (20 marks) Graphically represent the terms of the given sequence by graphing the functions $f(x) = x$ and $g(x) = \frac{x-1}{x+3}$ on the same coordinate system and precisely sketch the intersection of the two curves. Is the sequence bounded? Is it monotonous? Prove your answers with induction.

b) (10 marks) Is the given sequence convergent? If yes, calculate its limit $\lim_{n \to \infty} a_n$. Does your answer to the last question change if the initial term a_1 is any other non-negative number? Briefly explain why. What about if $a_1 = -2$? (A hint to answering the last question: Calculate the first few terms of the sequence.)

Question 2 (30 marks)

The temperature in the plane is given by the function $T\colon \mathbb{R}^2 \to \mathbb{R}$,

$$T(x, y) = e^{-3x^2 - 3y^2 + 6}.$$

a) (15 marks) Write the equations of the level curves for the levels $c = 1$ and $c = e^3$ and plot both in the same coordinate system.

b) (5 marks) Find the temperature gradient $(\operatorname{grad} T)(x, y)$.

c) (10 marks) We move from the point $P(1, 1)$ by a very small amount in the direction of the vector $(4, -3)$. Will the temperature increase or decrease?

Question 3 (30 marks)

The functions f and g are defined as

$$\begin{aligned} f(x) &= x(x - \pi) \\ g(x) &= \sin(x)e^{\cos(x)} \end{aligned}$$

a) (8 marks) Compute the indefinite integral of the function f.

b) (14 marks) Compute the indefinite integral of the function g.

c) (8 marks) Compute the area of the bounded region between the functions f and g.
If the result contains for instance e or π leave the expression as it is.
Some advice: To find the solutions to the equation $f(x) = g(x)$ in order to determine the interval of integration it suffices to consider the zeros of the functions f and g.