· Name and surname: ________________________ ID: ___________________

Second exam for OMA, 02.02.2020

- Time limit: **30 minutes**
- For a passing grade you have to achieve at least 50% of all points. The number in the bracket $[\cdot]$ tells you how many points you get for a correct solution to the question.
- Any attempt of copying the solution of someone else, talking, using electronic equipment is **strictly** forbidden.

1. [**40 points**] In this task let S be the **fourth nonzero** cypher of your student ID (vpisna številka), counted from the left to the right.

(a) Write down an example of the sequence $(a_n)_{n\in\mathbb{N}}$ consisting only of positive terms, such that the series $\sum_{n=1}^{\infty} a_n^{S+1}$ diverges, while the alternating series $\sum_{n=1\infty} (-1)^n a_n^{S+1}$ converges.

(b) Let $\sum_{n\in\mathbb{N}} a_n$ be a series consisting of only positive terms, such that $\lim_{n\to\infty} a_n = 1$ holds. Is it possible that with the changes of all terms a_{2n} (i.e., terms with even indices), the series becomes convergent?

(c) Let $(a_n)_n$ be a sequence of real numbers, such that $a_n \in [0,1]$ and $\lim_{n\to\infty} a_n = \frac{1}{2021}$.

i. Write down or draw an example of a function (not neccessarily continuous) $f : [0, S) \to \mathbb{R}$ such that $\lim_{n\to\infty} f(a_n) = S$ and $\lim_{x\uparrow S} f(x) = \infty$.

ii. Does there exist a continuous function $f : [0, 1] \to \mathbb{R}$ satisfying $\lim_{n\to\infty} f(a_n) = \infty$?

2. [**30 points**] In this task let T be the **third nonzero** cypher of your student ID (vpisna številka), counted from the right to the left.

(a) Write down an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ of two variables, which grows the most rapidly along the direction $(2, 1)$ in the point $(1, T)$.

(b) Write down an example of a function $g : \mathbb{R}^2 \to \mathbb{R}$ of two variables, which has a local minimum in the point $(1, T)$.

Hint: The Taylor polynomial of degree 2 of a function g in the point (x_0, y_0) could be helpful:

$$
\begin{aligned}
T_2(x,y) &= g(x_0,y_0) + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0) \\
&\quad + \frac{1}{2}g_{xx}(x_0,y_0)(x-x_0)^2 + g_{xy}(x_0,y_0)(x-x_0)(y-y_0) + \frac{1}{2}g_{yy}(x_0,y_0)(y-y_0)^2.
\end{aligned}
$$

(c) Let $\mathcal{C}$ be the curve, determined by the equation $h(x, y) = 0$. Assume that the point $(1, T)$ is an extremum of your function f from (2a) over the curve $\mathcal{C}$. Determine the direction of the tangent to the curve $\mathcal{C}$ in the point $(1, T)$.

3. **[30 points]**

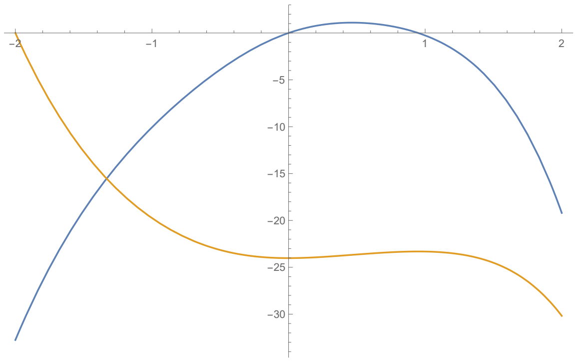

Image 1

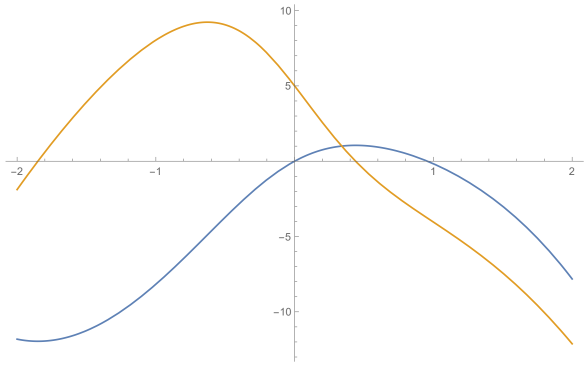

Image 2

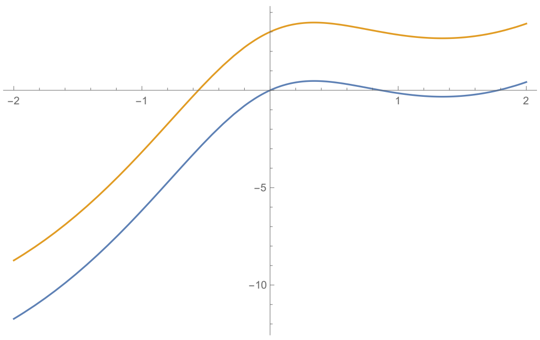

Image 3

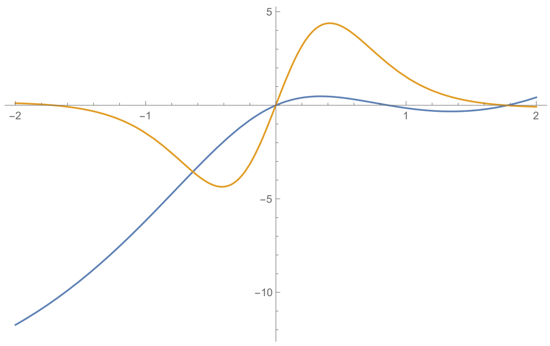

Image 4

For each of the following statements exactly one of the images above is appropriate (for each statement another image). Choose the appropriate image and **justify** your choice by finding the property of the image, which holds only for this image.

(a) On the image there are graphs of two indefinite integrals of some function $f : [-2, 2] \to \mathbb{R}$.

(b) On the image there are graphs of the function $f : [-2, 2] \to \mathbb{R}$ and its definite integral $F(x) = \int_{-2}^{x} f(t)\, dt$.

(c) On the image the **blue** graph represent the function $f : [-2, 2] \to \mathbb{R}$, while the orange one its derivative.