2. popravni kolokvij iz Osnov matematične analize
(Ljubljana, 14. februar 2017)

Time limit: 90 minutes. Every exercise is worth the same amount of points. Read the instructions carefully. You may use two A4 sheets with formulas. The results will be published on `ucilnica.fri.uni-lj.si`.

Justify all your answers!

1. An integer sequence $(a_n)_{n\in\mathbb{N}}$ is defined with initial terms $a_0 = 0$, $a_1 = 2$ and a recurrence formula
$$a_n = 4a_{n-1} - 3a_{n-2},$$
which applies for $n \geq 2$. Show, using mathematical induction, that for every $n \in \mathbb{N}$ we have
$$a_n = 3^n - 1.$$

2. Calculate the following limits, if they exist:
   (a) $\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{x-1}$,
   (b) $\lim\limits_{x\to\infty} \frac{e^x+x^2}{e^x+4x}$,
   (c) $\lim\limits_{x\searrow 2} \frac{x^2+4x-12}{|x-2|}$ in $\lim\limits_{x\nearrow 2} \frac{x^2+4x-12}{|x-2|}$.

3. (a) Find all points in the plane, such that $y = \frac{1}{x}$ and the function
$$f(x,y) = -\frac{1}{x} + \frac{1}{2}\ln y$$
has an extreme.
   (b) For each of these points find the directional derivative in the direction of the gradient.

4. Given the function $f(x) = (x^2-1)e^x$
   (a) plot its graph,
   (b) calculate the indefinite integral
$$\int (x^2-1)e^x\, dx$$
and
   (c) determine the area, delimited by the graph and the x-axis.

Justify your answers!