Computational topology - group project

Generating Labyrinths

Introduction: While generating labyrinths, the architect has to make sure there are no unreachable chambers. In this project you will develop various labyrinth generating models in 2D and 3D while keeping track of the unreachable chambers.

A detailed description of the 2D setting: Decide on a preferred triangulation of a planar region. One option would be a square grid of initial size 10×10.

1. Generate labyrinths in a plane using three different approaches:
   (a) by randomly adding edges one by one;
   (b) by randomly walking along the 1-skeleton of the chosen triangulation and adding the visited edges. Random walks should start at the boundary of the region;
   (c) by starting with the 1-skeleton of a full triangulation and then randomly erasing edges.
2. Approaches (a) and (b) should be pursued in two different modes:
   - constrained mode: edges are added only if they create no unreachable chamber. Do the labyrinths constructed using (a) look different from those using (b)?
   - unconstrained mode: edges are added in any case. Experimentally analyze the average number of obtained unreachable chambers as a function of the number of added edges. Is there a difference between (a) and (b)?
3. Approach (c): the erasing procedure should proceed randomly until there are only three unreachable chambers left. After that you should erase only three more edges to eliminate all unreachable chambers.

A detailed description of the generalized 2D setting: Extend your approach to Torus and Klein bottle. In both cases generate labyrinths with non-zero H_1 encompassing no unreachable regions. This can be achieved by admitting two loops in the labyrinth, which are homologous to the generators of $H_1(T)$.

A description of the 3D setting: Try adjusting any of the above approaches to generate a connected labyrinth in 3D with no unreachable chambers. Interpret H_1 in this case.

Results: The report should include a description of generating models, a pseudocode, methods of computation, results of experiments, generated 2D labyrinths, and division of work.

Students are encouraged to take the initiative and possibly implement their own ideas on the theme of the project: perhaps thinking of their own way to generate the labyrinth, etc.