Computational topology - group project

Topological optimization

Introduction: Persistent homology via Rips complexes for finite subsets in $\mathbb{R}^2$ takes as an input a pointcloud in the plane and outputs a barcode. This assignment is stable (i.e., continuous), meaning that small perturbations in a pointcloud result in small changes in barcodes. Now take a single bar in the obtained barcode. In this project you will explore how the pointcloud needs to change in order to prolong (or shorten) the chosen bar. The procedure will reveal topologically optimal configurations of points.

Goal: Deform a pointcloud to obtain a desired persistence diagram.

Methodology: In order to increase the birth of a 1-dimensional bar, or decrease the death of a 0-dimensional bar, move the pair of vertices representing the relevant critical simplex closer to each other. To prolong the death of a 1-dimensional bar, the three vertices representing the critical simplex should move away from each other. Alternatively, a formula can be found in Lemma 2 of `https://arxiv.org/pdf/2309.08241`. For more details you can also check a PhD Thesis `https://qmro.qmul.ac.uk/xmlui/handle/123456789/96149` or a paper `https://inria.hal.science/hal-02969305/file/_ICML21__Optimizing_persistent_homology_based_functions.pdf`. In order for the optimization to make sense, restrict your pointcloud to a bounded domain. You can choose one or more of the following: a square, a circle, a torus, a sphere, etc. You can use any software to compute persistent homology. However, you should implement the optimization (i.e., the movement) of the pointcloud yourself.

Tasks: The described approach can be tested in two settings.

1. Start with a fixed pointcloud. Compute its barcode. Then experiment with your approach along the following lines:
   - Take one or more of the longest 0-dimensional bars and try to decrease their deaths.
   - Take one or more of the longest 1-dimensional bar and try to increase/decrease their births or deaths. What do you get if you only prolong the longest 1-dimensional bar, and shorten all other 1-dimensional bars?
2. Start with a fixed pointcloud A. Compute its barcode PD(A). Now generate a pointcloud B in the chosen region. Keep deforming B by transforming its persistence diagram towards PD(A). Compare the resulting pointcloud with A.

Results: The report should include a description of methodology and tasks undertaken, a pseudocode, methods of computation, results of experiments, and division of work.

Students are encouraged to take the initiative and possibly implement their own ideas on the theme of the project: perhaps thinking of their own topological optimization setting, etc.