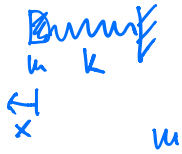
NIHANJE IN VALOVANJE

1) SISTEM

Stanje, enačbe gibanje $F = ma$ (Newton)

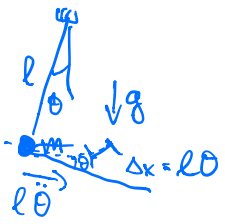
Vzmetno nihalo



Hookov zakon: $F = -kx$

$$ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} = -kx \qquad \boxed{\ddot{x} + \frac{k}{m}x = 0}$$

Matematično nihalo



$$ml\ddot{\theta} = -mg\sin\theta \qquad \boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$$

2) NIHANJE $x(t) = x(t+T)$

perioda T [s]

Prehajanje energije: kinetična ↔ potencialna

$\nu = \frac{1}{T}$ [Hz] frekvenca $\quad \omega = 2\pi\nu$ krožna frekvenca

angl. angular frequency

Harmonično nihanje $x(t) = A\sin(\omega t + \varphi)$

A amplituda, $\omega t + \varphi$ faza, φ fazni zamik

$\sin(x + 2\pi) = \sin x$

$\omega T = 2\pi \quad \boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$

dušenje $x(t) = Ae^{-t/\tau}\sin(\omega t + \varphi)$

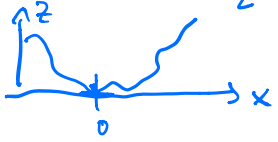
τ relaksacijski čas $\quad 5\tau \quad 1\%$

vzbujanje

3) LINEARNOST

Taylorjev izrek $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3\cdot 2}x^3 + \dots$

$F = ma \quad F = -\frac{dU}{dx} \quad U(x)$



$U = mgz(x)$

$U = \frac{k}{2}x^2 + \dots \quad F = -\frac{dU}{dx} = -kx \rightarrow$ linearizacija

$\ddot{\theta} + \frac{g}{l}\sin\theta = 0 \qquad \sin\theta = \theta - \frac{\theta^3}{6} + \dots$

4) EULERJEVA ENAČBA $e^{i\theta} = \cos\theta + i\sin\theta$ $\quad N! = N\cdot(N-1)\cdots 2\cdot 1$

$$e^{i\theta} = 1 + i\theta - \frac{1}{2}\theta^2 - i\frac{1}{3!}\theta^3 + \frac{1}{4!}\theta^4 + \ldots = \underbrace{\left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} + \ldots\right)}_{\cos\theta} + i\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots\right)}_{\sin\theta}$$

$x \qquad z = x + iy$

$$\left.\begin{aligned} &\ddot{x} + \frac{k}{m}x = 0 \\ &i\ddot{y} + i\frac{k}{m}y = 0 \end{aligned}\right\} \quad \frac{d^2}{dt^2}(x+iy) + \frac{k}{m}(x+iy) = 0 \qquad \ddot{z} + \frac{k}{m}z = 0$$

$x = a\sin(\omega t) + b\cos(\omega t)$ $\qquad z = \underline{A e^{i\omega t}}$

Polarni zapis $\qquad x = \mathrm{Re}\, z = \mathrm{Re}\left[z e^{i\omega t}\right]$

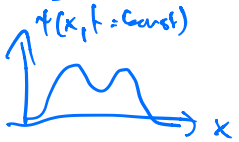
$z = A e^{i\omega t} \qquad A = |A| e^{i\varphi} = x_0 e^{i\varphi}$

$z = x_0 e^{i\varphi} e^{i\omega t} \qquad x = \mathrm{Re}\left[x_0 e^{i\varphi} e^{i\omega t}\right] = x_0\, \mathrm{Re}\, e^{i(\omega t + \varphi)} = x_0\cos(\omega t + \varphi)$

Kompleksna amplituda $A = x_0 e^{i\varphi}$

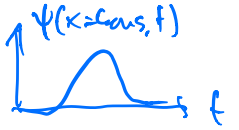
5) VALOVANJE, VALOVNA ENAČBA

$\psi(x,t)$ $\quad t = \text{konst.}$

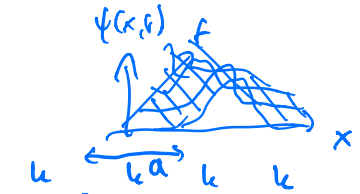


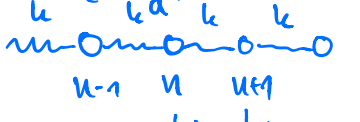
trenutna slika

$x = \text{konst.}$



časovni potek





masa m

$\psi_n(t) \qquad \psi_{n+1}(t)$

$\psi_{n+1} - \psi_n$

$$m\ddot{\psi}_n = k(\psi_{n+1} - \psi_n) + k(\psi_{n-1} - \psi_n)$$

$$m\ddot{\psi}_n = k(\psi_{n+1} - 2\psi_n + \psi_{n-1})$$

$\psi(x,t) \qquad \psi(x = na, t) = \psi_n(t)$

$\dot{f} = \frac{df}{dt} \quad f' = \frac{df}{dx}$

$\dot{f} = \frac{\partial f}{\partial t} \quad f' = \frac{\partial f}{\partial x}$

parcialni odvod

$$f' = \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, t) - f(x, t)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\frac{\Delta x}{2}, t) - f(x-\frac{\Delta x}{2}, t)}{\Delta x}$$

$$f'' = \frac{\partial^2 f}{\partial x^2} = \lim_{\Delta x \to 0} \frac{\frac{\partial f}{\partial x}(x+\Delta x/2, t) - \frac{\partial f}{\partial x}(x-\Delta x/2, t)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[f(x+\Delta x, t) - f(x, t)] - [f(x, t) - f(x-\Delta x, t)]}{\Delta x^2}$$

$$\psi_{n+1} - 2\psi_n + \psi_{n-1} = \psi(na+a) - 2\psi(na) + \psi(na-a) = \frac{\partial^2 \psi}{\partial x^2} a^2$$

$$m\ddot{\psi}_n = ka^2 \frac{\partial^2 \psi}{\partial x^2}$$

$$\psi_n = \psi(x = na)$$

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = \left(\frac{ka^2}{m}\right) \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

Val. enačba $\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}}$ → linearna

$\psi(x,t) = f(x-ct) + g(x+ct)$ splošna rešitev

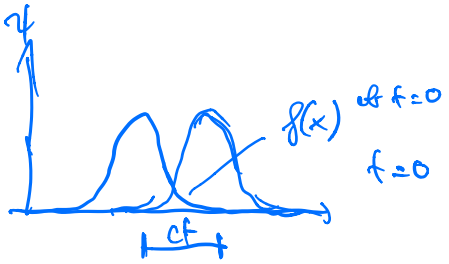
NAČELO SUPERPOZICIJE!

$\psi'' = f'' + g''$

$\frac{1}{c^2} \ddot{\psi} = \frac{1}{c^2} [(-c)^2 f'' + c^2 g''] = f'' + g''$ ✓

$\psi = f(x-ct)$

val v desno



$\psi = g(x+ct)$

val v levo

6) RAVNI VAL

$$\psi = A \sin[k(x - ct)] \qquad \frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\psi(x,t) = f(x) e^{-i\omega t}$$

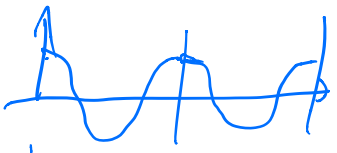
$$f''(x) e^{-i\omega t} = \frac{1}{c^2} f(x) (-i\omega)^2 e^{-i\omega t}$$

$$f'' + \frac{\omega^2}{c^2} f = 0 \qquad f = A e^{ikx}$$

$$A(ik)^2 e^{ikx} + \frac{\omega^2}{c^2} A e^{ikx} = 0$$

$$\frac{\omega^2}{c^2} = k^2 \qquad \boxed{\frac{\omega}{c} = k} \qquad \omega = 2\pi\nu = 2\pi/T$$

$$\psi(x,t) = A e^{i(kx - \omega t)} \longrightarrow \psi(x,t) = A \cos(kx - \omega t)$$



$$\frac{2\pi}{T} \frac{1}{c} = \frac{2\pi}{\lambda} \qquad \frac{\lambda}{T} = c$$

$$\boxed{c = \frac{\lambda}{T}}$$

k ... valovno število

λ ... valovna dolžina

$$k \cdot \lambda = 2\pi \qquad k = \frac{2\pi}{\lambda}$$