POSTULATI KVANTNE MEHANIKE

1) $|\psi\rangle$ keti so elementi velikega Hilbertovega prostora, $\langle\psi|\psi\rangle=1$

2) Opazljivke $\Leftrightarrow$ sebiadjungirani operatorji $\hat{A}^\dagger=\hat{A}$

3) Bornovo pravilo
$$\hat{A}=\begin{bmatrix}\lambda_1 & & \emptyset\\ & \lambda_2 & \\ \emptyset & & \ddots\ \lambda_n\end{bmatrix}=\sum_{i=1}^{n}|i\rangle\,\lambda_i\,\langle i|$$

$|i\rangle=\begin{pmatrix}0\\ \vdots\\ 1\\ \vdots\\ 0\end{pmatrix}$ Merimo da vrednost λ_i z verjetnostjo $p_i=|\langle i|\psi\rangle|^2$.

1) Vektorski prostor, $|a\rangle=\begin{bmatrix}a_1\\ \vdots\\ a_n\end{bmatrix}$ ← amplituda (verjetnostna) $\qquad \langle a|=[a_1^* \ \dots \ a_n^*]$

$$(|a\rangle,|b\rangle)=\langle a|b\rangle=\sum_{i=1}^{n}a_i^* b_i \qquad |a\rangle\perp|b\rangle, \text{ če } \langle a|b\rangle=0$$

$a_1=|a_1|e^{i\varphi_1}$ ← omogoča interferenco

↑ verjetnost

Kubita: $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$ $\alpha,\beta\in\mathbb{C}$

$$\langle\psi|\psi\rangle=1=(|\psi\rangle,|\psi\rangle)=[\alpha^* \ \beta^*]\cdot\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=|\alpha|^2+|\beta|^2=1$$

2) op $\Leftrightarrow$ matrika $A_{ij}=\langle i|\hat{A}|j\rangle \qquad \hat{A}\Leftrightarrow\begin{bmatrix}A_{11} & A_{12} & \\ A_{21} & \ddots & \\ & & A_{nn}\end{bmatrix}$

$$\hat{A}=\sum_{ij}|i\rangle A_{ij}\langle j| \qquad \mathbb{1}=\begin{pmatrix}1 & & \emptyset\\ & 1 & \\ \emptyset & & 1\end{pmatrix}=\sum_{i=1}^{n}|i\rangle\langle i|=\sum_{i=1}^{n}|e_i\rangle\langle e_i|$$

$\langle e_i|e_j\rangle=\delta_{ij}=\begin{cases}1 & \text{če } i=j\\ 0 & \text{sicer}\end{cases}$

Kroneckerjev simbol δ

$$\hat{A}=\mathbb{1}\hat{A}\mathbb{1}=\sum_{i=1}^{n}|i\rangle\langle i|\,\hat{A}\sum_{j=1}^{n}|j\rangle\langle j|$$
$$=\sum_{i,j=1}^{n}|i\rangle\underbrace{\langle i|\hat{A}|j\rangle}_{\substack{\text{število}\\ A_{ij}}}\langle j| \Rightarrow \overset{\text{operator}}{\hat{A}}=\sum_{ij}|i\rangle\overset{\text{repr.}}{A_{ij}}\langle j|$$

odvisno od baze !!!!!

←

$\hat{B}\hat{A}|\psi\rangle\neq\hat{A}\hat{B}|\psi\rangle$ komutator $[\hat{A},\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}$

antikomutator $\{\hat{A},\hat{B}\}=\hat{A}\hat{B}+\hat{B}\hat{A}$

$$\langle a|\hat{A}|b\rangle=(\langle b|\hat{A}^\dagger|a\rangle)^* \qquad \underbrace{\langle a|\hat{A}^\dagger|b\rangle=(\langle b|\hat{A}|a\rangle)^*}_{[A^\dagger]_{ij}=A_{ji}^*}$$

Paulijeve matrike: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$[\sigma_x, \sigma_y] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i\sigma_z$$

$[\sigma_y, \sigma_z] = 2i\sigma_x$ $\quad [\sigma_z, \sigma_x] = 2i\sigma_y$

$\hat{P}_i = |i\rangle\langle i| \rightarrow$ projektor na vektor $|i\rangle$ $\quad \hat{P}_i |i\rangle = |i\rangle \underbrace{\langle i|i\rangle}_{1} = |i\rangle$

$j \neq i \quad \hat{P}_i |j\rangle = |i\rangle \underbrace{\langle i|j\rangle}_{0} = 0$

$$|a\rangle = \sum_{j=1}^{n} a_j |j\rangle$$

$$\hat{P}_i |a\rangle = \hat{P}_i \left(\sum_{j=1}^{n} a_j |j\rangle \right) = \sum_{j=1}^{n} a_j \underbrace{\hat{P}_i |j\rangle}_{\delta_{ij}|i\rangle} = a_i |i\rangle$$

$\hat{P}_i^2 = \hat{P}_i$ $\quad \hat{P}_i \hat{P}_j = 0 \;\; i \neq j$ $\quad \hat{P}_i \hat{P}_j = \delta_{ij} \hat{P}_i$

$\hat{A}|v\rangle = \lambda |v\rangle$ $\quad$ 1) $\hat{A} = \hat{A}^\dagger \rightarrow \lambda \in \mathbb{R}$

2) $\lambda_1 \neq \lambda_2 \rightarrow \langle v_1 | v_2 \rangle = 0$

$\hat{A} = \sum_{i=1}^{n} |i\rangle \lambda_i \langle i|$ če velja $\hat{A}|i\rangle = \lambda_i |i\rangle$

↑ lastni vektorji

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0$

$\boxed{\lambda_{1,2} = \pm 1}$

$\lambda = 1: \; \sigma_x |v\rangle = |v\rangle$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_2 \\ v_1 \end{pmatrix}$

$v_1 = v_2$

$\boxed{|x, +1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}$

$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\begin{vmatrix} -\lambda & -i \\ i & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0$

$\boxed{\lambda_{1,2} = \pm 1}$

$\lambda = 1: \; \sigma_y |v\rangle = |v\rangle$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$-i v_2 = v_1 \quad i v_1 = v_2$

$\boxed{|y, +1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}}$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = -(1-\lambda)(1+\lambda) = 0$

$\boxed{\lambda_{1,2} = \pm 1}$

$\lambda = 1: \; \sigma_z |v\rangle = |v\rangle$

$\boxed{|z, +1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$\lambda=-1$: $\sigma_x|v\rangle=-|v\rangle$

$\begin{pmatrix}v_2\\v_1\end{pmatrix}=-\begin{pmatrix}v_1\\v_2\end{pmatrix}$

$v_1=-v_2$

$$\boxed{|x,-1\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1/\sqrt2\\-1/\sqrt2\end{pmatrix}}$$

$\lambda=-1$: $\sigma_y|v\rangle=-|v\rangle$

$$\boxed{|y,-1\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\-i\end{pmatrix}=\begin{pmatrix}1/\sqrt2\\-i/\sqrt2\end{pmatrix}}$$

$\lambda=-1$: $\sigma_z|v\rangle=-|v\rangle$

$\begin{bmatrix}v_1\\-v_2\end{bmatrix}=-\begin{bmatrix}v_1\\v_2\end{bmatrix}$

$v_1=0 \quad v_2=1$

$$\boxed{|z,-1\rangle=\begin{pmatrix}0\\1\end{pmatrix}}$$

Bornovo pravilo: λ_i so možni rezultati, $p_i=|\langle i|\psi\rangle|^2$.

Pogoj: $\langle\psi|\psi\rangle=1 \qquad \langle i|j\rangle=\delta_{ij}$

Primer 1: $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle \qquad p_0=|\alpha|^2$

$\hat P_0=|0\rangle\langle 0| \qquad \lambda=1,0 \qquad |0\rangle$ lastni vektor

$\lambda=1$ z verjetnostjo $p_0=|\langle 0|\psi\rangle|^2=|\alpha|^2$ ✓

$\hat P_1=|1\rangle\langle 1| \qquad |1\rangle$

$p_1=|\langle 1|\psi\rangle|^2=|\beta|^2$ ✓

Primer 2: $\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad |\psi\rangle=\alpha|0\rangle+\beta|1\rangle$

$\lambda=1 \qquad p_{+1}=\left|\begin{bmatrix}1/\sqrt2 & 1/\sqrt2\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix}\right|^2=\left|\frac{\alpha+\beta}{\sqrt2}\right|^2=\frac{|\alpha+\beta|^2}{2}$

$\lambda=-1 \qquad p_{-1}=\left|\begin{bmatrix}1/\sqrt2 & -1/\sqrt2\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix}\right|^2=\left|\frac{\alpha-\beta}{\sqrt2}\right|^2=\frac{|\alpha-\beta|^2}{2}$

$$p_{+1}+p_{-1}=\frac12\left(|\alpha+\beta|^2+|\alpha-\beta|^2\right)=\frac12\left((\alpha^*+\beta^*)(\alpha+\beta)+(\alpha^*-\beta^*)(\alpha-\beta)\right)$$

$$=\frac12\left(|\alpha|^2+|\beta|^2+\cancel{\alpha^*\beta}+\cancel{\alpha\beta^*}+|\alpha|^2+|\beta|^2-\cancel{\alpha^*\beta}-\cancel{\alpha\beta^*}\right)$$

$$=|\alpha|^2+|\beta|^2=1 \checkmark$$

Pričakovana vrednost: $\langle\hat A\rangle=\sum_{i=1}^n\lambda_i p_i=\sum_{i=1}^n\lambda_i|\langle i|\psi\rangle|^2$

$$=\sum_{i=1}^n\lambda_i\underbrace{\langle i|\psi\rangle^*}_{\langle\psi|i\rangle}\langle i|\psi\rangle$$

$$\langle\hat A\rangle=\langle\psi|\hat A|\psi\rangle$$

$$=\sum_{i=1}^n\langle\psi|i\rangle\lambda_i\langle i|\psi\rangle=\langle\psi|\hat A|\psi\rangle$$

$$\hat A=\sum|i\rangle\lambda_i\langle i|$$

1. Faze verjetnostnih amplitud

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = |\alpha|e^{i\varphi_\alpha}|0\rangle + |\beta|e^{i\varphi_\beta}|1\rangle$$

$$|\psi_1\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle \qquad p_0 = \left(\frac{3}{5}\right)^2 = \frac{9}{25} = 0{,}36$$

$$|\psi_2\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle \qquad p_1 = \left(\frac{4}{5}\right)^2 = \frac{16}{25} = 0{,}64$$

$$\sigma_x \quad \lambda_{1,2} = \pm 1 \quad |1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \longleftarrow p_{+1} = \frac{|\alpha+\beta|^2}{2}$$

$$|-1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \longleftarrow p_{-1} = \frac{|\alpha-\beta|^2}{2}$$

$$|\psi_1\rangle \longrightarrow p_{+1} = \frac{(7/5)^2}{2} = \frac{49}{50} \qquad p_{-1} = \frac{(1/5)^2}{2} = \frac{1}{50}$$

$$|\psi_2\rangle \longrightarrow p_{+1} = \frac{1}{50} \qquad p_{-1} = \frac{49}{50}$$

$\varphi_\alpha, \varphi_\beta$ pomembno !!!

$$|z_1 z_2| = |z_1||z_2|$$

$$p_i^{(\psi)} = |\langle i|\psi\rangle|^2 \qquad |\psi\rangle \longrightarrow |\varphi\rangle = e^{i\gamma}|\psi\rangle$$

$$p_i^{(\varphi)} = |\langle i|\varphi\rangle|^2 = |\langle i|e^{i\gamma}|\psi\rangle|^2 = |e^{i\gamma}\langle i|\psi\rangle|^2$$

$$= |e^{i\gamma}|^2|\langle i|\psi\rangle|^2 = |\langle i|\psi\rangle|^2 = p_i^{(\psi)}$$

Pri vseh meritvah dobimo za $|\psi\rangle$ in $|\varphi\rangle = e^{i\gamma}|\psi\rangle$ povsem identične rezultate. Globalna faza stanja ni pomembna (ni merljiva).

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = |\alpha|e^{i\varphi_\alpha}|0\rangle + |\beta|e^{i\varphi_\beta}|1\rangle$$

$$= e^{i\varphi_\alpha}\left(|\alpha|\,|0\rangle + |\beta|\,e^{i(\varphi_\beta-\varphi_\alpha)}|1\rangle\right)$$

$$= e^{i\varphi_\alpha}\left(|\alpha|\,|0\rangle + |\beta|\,e^{i\varphi}|1\rangle\right)$$

$\varphi = \varphi_\beta - \varphi_\alpha$ merljivo!

Fazne razlike so merljive.

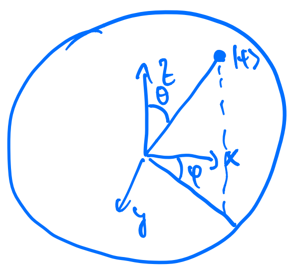
2. Blochova sfera

$$|\psi\rangle = |\alpha|\,|0\rangle + |\beta|\,e^{i\varphi}\,|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1 \qquad |\alpha| = \cos\theta/2 \qquad \cos^2\theta/2 + \sin^2\theta/2 = 1$$

$$|\beta| = \sin\theta/2$$

$$|\psi\rangle = \cos\theta/2\,|0\rangle + \sin\theta/2\,e^{i\varphi}\,|1\rangle$$



φ, θ : koordinati v sferičnem koordinatnem sistemu

$z,+1$ $\quad |\psi\rangle = |0\rangle \rightarrow$ $\theta = 0$, φ poljuben severni pol

$z,-1$ $\quad |\psi\rangle = |1\rangle \rightarrow$ $\theta = \pi$, φ poljuben južni pol

$x,+1$ $\quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \rightarrow \theta = \pi/2, \ \varphi = 0$ točka na ekvatorju pri $\varphi = 0$

$x,-1$ $\quad \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \rightarrow \theta = \pi/2, \ \varphi = \pi$ —||— pri $\varphi = \pi$

$y,+1$ $\quad \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \rightarrow \theta = \pi/2, \ \varphi = \pi/2$ —||— pri $\varphi = \pi/2$

$y,-1$ $\quad \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \rightarrow \theta = \pi/2, \ \varphi = -\pi/2$ —||— pri $\varphi = -\pi/2$

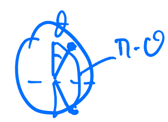
$$|\psi\rangle = \cos\theta/2\,|0\rangle + \sin\theta/2\,e^{i\varphi}|1\rangle$$

$$|\psi'\rangle = \cos\left(\tfrac{\pi}{2} - \tfrac{\theta}{2}\right)|0\rangle + \sin\left(\tfrac{\pi}{2} - \tfrac{\theta}{2}\right)e^{i(\varphi+\pi)}\,|1\rangle$$

$$= \sin\theta/2\,|0\rangle - \cos\theta/2\,e^{i\varphi}\,|1\rangle$$

$\theta \rightarrow \pi - \theta$

$\varphi \rightarrow \varphi + \pi$ $\leftarrow$



$$\langle\psi|\psi'\rangle = cs - s\,\cancel{e^{-i\varphi}}\,c\,\cancel{e^{i\varphi}} = 0$$

Diametralno nasprotne točke pripadajo ortogonalnim vektorjem.

3. Kolaps valovne funkcije

POSTOLAT 4: Takoj po meritvi, ko izmerimo λ_i, je sistem v stanju $|\psi(\text{po meritvi})\rangle = |i\rangle$.

Meritev je destruktivna. Ni determinizma.

Kopenhagenska interpretacija.

(4.) Izrek o prepovedi kloniranja

Wooters, Zurek (1982)

POSTULATI ZA KVANTNO INFORMACIJO:

1) velja načelo superpozicije. $\alpha|0\rangle + \beta|1\rangle$

2) Branje destruktivno.

3) Kopiranje ni možno.

$$A^T B^T = (BA)^T$$
$$A^\dagger B^\dagger = (BA)^\dagger$$

(5.) Unitarnost

$$\hat{U}\hat{U}^\dagger = \hat{U}^\dagger \hat{U} = \mathbb{1} \qquad |\psi\rangle \qquad |\varphi\rangle \qquad \langle\psi|\varphi\rangle$$

$$|\psi'\rangle = \hat{U}|\psi\rangle \qquad |\varphi'\rangle = \hat{U}|\varphi\rangle \qquad \langle\psi'|\varphi'\rangle = \langle\psi|\underbrace{\hat{U}^\dagger \hat{U}}_{\mathbb{1}}|\varphi\rangle = \langle\psi|\varphi\rangle$$

$$\downarrow$$

$$\langle\psi'| = \langle\psi|\hat{U}^\dagger$$

$\hat{U}$ ohranja skalarni produkt.

Ortogonalnost: $\hat{O}\hat{O}^T = \hat{O}^T\hat{O} = \mathbb{1}$ $\qquad \hat{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \rightarrow$ rotacija za kot θ

POSTULAT 5: Časovni potek opisuje neka unitarna transformacija.

$$|\psi(t_2)\rangle = \hat{U}\,|\psi(t_1)\rangle \qquad t_2 \geq t_1$$

$$\hat{U}(t_2, t_1)$$

$$\langle\psi(t_1)|\psi(t_1)\rangle = 1 \quad \Rightarrow \quad \langle\psi(t_2)|\psi(t_2)\rangle = 1$$

$$\hat{U}\hat{U}^\dagger = \mathbb{1} \qquad \boxed{\hat{U}^\dagger = \hat{U}^{-1}} \qquad AA^{-1} = \mathbb{1} \qquad |\psi(t_1)\rangle = \hat{U}^{-1}|\psi(t_2)\rangle$$

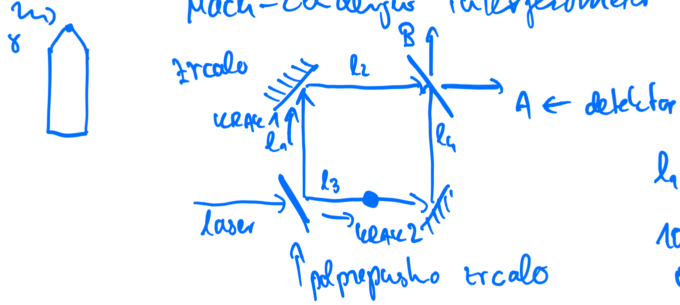
$$|\psi(t_1)\rangle = U^\dagger\,|\psi(t_2)\rangle$$

Kvantna mehanika (časovni potek) je reverzibilna. (Meritve pa niso!!)

$|\lambda| = 1 \qquad |\psi\rangle = \hat{U}|\psi\rangle \rightarrow$ izraža simetrijo.

6. Preizkuševalec bomb

Mach-Zehnderjev interferometer



l_1+l_2 vs. l_3+l_4

100% na detektor A
0% na detektor B

1) Bomba je pokvarjena. Vsi fotoni padejo na detektor A.

$|\psi\rangle = 1/\sqrt{2}\,(|\gamma \text{ v kraku } 1\rangle + |\gamma \text{ v kraku } 2\rangle)$

2) Bomba deluje.
- a) → γ v kraku 2, bomba eksplodira
- b) → γ v kraku 1, bomba ne eksplodira

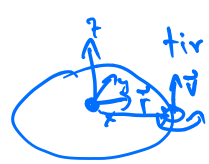
Kolaps valovne funkcije.

$|\psi'\rangle = |\gamma \text{ v kraku } 1\rangle$

- i) γ zaznamo z detektorjem A 50%
- ii) γ zaznamo z detektorjem B 50% → Bomba deluje

1/4 vseh delujočih.

7. Spin



tir → tirna vrtilna količina $\vec{\Gamma} = \vec{r} \times (m\vec{v})$

Γ_z

$\updownarrow$

$\vec{\mu} = \frac{1}{2} e\, \vec{r} \times \vec{v}$ ← naboj delca

$\vec{\mu} = \frac{1}{2} \frac{e}{m} \vec{\Gamma}$
magnetni moment

lastna vrtilna količina, $\Gamma_z = \frac{\hbar}{2}$ ali $-\frac{\hbar}{2}$

$S = 1/2 \quad S_z = -1/2 \quad S_z = +1/2$

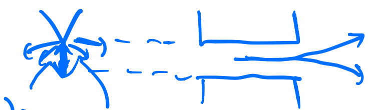
S celo ali polcelo število, $S = 0, 1/2, 1, 3/2, \dots$

$S_z = -S, -S+1, \dots, S-1, S$

$$\vec{\mu}_e = -\frac{e_0}{2m_e} g_s \vec{\Gamma} \qquad g_s = 2{,}00231930419922\,(1 \pm 1{,}5\cdot 10^{-12})$$

8. Eksperiment Sterna in Gerlacha

$\vec{\mu}$ $\qquad U = \vec{\mu}\cdot\vec{B}$



$$\mu_z \in \left[-\frac{e_0 g_s \hbar}{2m_e\, 2}, +\frac{e_0 g_s \hbar}{2m_e\, 2}\right]$$

klasično:

dejansko (K.M.):

1922

spin je kvantiziran!!

$|\uparrow\rangle, |\downarrow\rangle$

$|\uparrow\rangle$

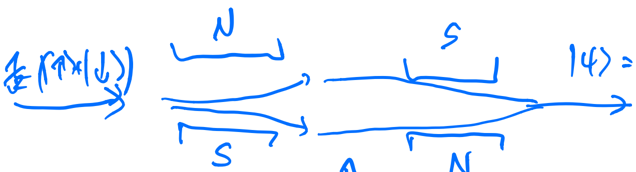
$|\downarrow\rangle$

$|+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

$|+\rangle$

50% ali 50%

$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$



$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

ne pomerimo