AKSIOMI

1) Stanje $|\psi\rangle$ je element Hilbertovega prostora. $\langle\psi|\psi\rangle = 1$
2) Opazljivke so opisane s Hermitskimi operatorji. $A = A^\dagger$
3) Bornovo pravilo: $\hat{A} = \sum_{i=1}^{n} \underbrace{\lambda_i}_{\text{lastna vrednost}} \underbrace{|i\rangle\langle i|}_{\text{lastni vektor}}$ $\qquad \langle i|j\rangle = 0$ če $\lambda_i \neq \lambda_j$

   Pri meritvi količine A dobimo rezultat λ_i z verjetnostjo $|\langle i|\psi\rangle|^2$.
4) Meritve so destruktivne. Če λ_1, potem je sistem takoj po meritvi v stanju $|1\rangle$.
5) Časovni potek opisuje neka unitarna matrika, $UU^\dagger = \mathbb{1} = U^\dagger U$.

Schrödingerjeva enačba

$$|\psi(t_2)\rangle = U(t_2, t_1)\,|\psi(t_1)\rangle \qquad |\psi(t_3)\rangle = U(t_3, t_2)\,|\psi(t_2)\rangle$$

$$|\psi(t_3)\rangle = U(t_3, t_2)\,|\psi(t_2)\rangle = U(t_3, t_2)\,U(t_2, t_1)\,|\psi(t_1)\rangle$$
$$= U(t_3, t_1)\,|\psi(t_1)\rangle$$

$$U(t_3, t_1) = U(t_3, t_2)\,U(t_2, t_1) \qquad t_3 \geq t_2 \geq t_1$$

$$U(t+\Delta t, t) \longrightarrow |\psi(t+\Delta t)\rangle = U(t+\Delta t, t)\,|\psi(t)\rangle$$

$\Delta t \to 0 \qquad U(t,t) = \mathbb{1} \qquad \hat{U}(t+\Delta t, t) = \mathbb{1} + \Delta t\,\hat{K}$

$$\mathbb{1} = \begin{pmatrix} 1 & & \emptyset \\ & \ddots & \\ \emptyset & & 1 \end{pmatrix}$$

$U \to U_{ij} = \langle i|\hat{U}|j\rangle \qquad U_{ij}(t+\Delta t, t) = \delta_{ij} + \Delta t\, K_{ij}$

$$\mathbb{1}_{ij} = \delta_{ij} = \begin{cases} 1 & i=j \\ 0 & \text{sicer} \end{cases}$$

$U^\dagger U = \mathbb{1} \quad \rightarrow \sum_j \underbrace{(U^\dagger)_{ij}}_{U^*_{ji}} U_{jk} = \delta_{ik}$

$$\sum_j (\delta_{ij} + \Delta t\, K^*_{ji})(\delta_{jk} + \Delta t\, K_{jk})$$

$$\approx \sum_j \left(\delta_{ij}\delta_{jk} + \Delta t\,\delta_{ij} K_{jk} + \Delta t\,\delta_{jk} K^*_{ji}\right) = \delta_{ik}$$

$$\delta_{ik} + \Delta t\, K_{ik} + \Delta t\, K_{ki}^* = \delta_{ik} \quad \forall i,k$$

$$\sum_j \delta_{ij}\delta_{jk} = \begin{cases} 1 & i=k \\ 0 & i\neq k \end{cases} = \delta_{ik}$$

$\delta_{ij}\delta_{ji}$ $\quad \delta_{1j}\delta_{j2}$

$$\sum_{j=1} \delta_{ij}^2 = 1$$

$$K_{ik} + K_{ki}^* = 0$$

$$K_{ik} = -\frac{i}{\hbar} H_{ik}$$

$$H_{ik} - H_{ki}^* = 0 \Rightarrow \hat{H} = \hat{H}^\dagger$$

$$\hat{K} = -i/\hbar\, \hat{H}$$

$$\hat{U}(\Delta t) = 1\!\!1 - i/\hbar\, \hat{H}\,\Delta t$$

↑ Hamiltonka ali Hamiltonov operator

$$|\psi(t+\Delta t)\rangle = \hat{U}|\psi(t)\rangle = \underbrace{|\psi(t)\rangle}\, - i/\hbar\,\Delta t\,\hat{H}\,|\psi(t)\rangle$$

$$i\hbar\,\frac{|\psi(t+\Delta t)\rangle - |\psi(t)\rangle}{\Delta t} = \hat{H}\,|\psi(t)\rangle$$

$$\boxed{i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}\,|\psi(t)\rangle}$$

Schrödingerjeva enačba

$$\hat{H}|i\rangle = \lambda_i |i\rangle \qquad |\psi(t)\rangle = e^{-ict}|i\rangle$$

$$|\psi(t=0)\rangle = |i\rangle \qquad i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

$$i\hbar(-ic)\cancel{e^{-ict}}|i\rangle = \lambda_i \cancel{e^{-ict}}|i\rangle$$

$$\hbar c = \lambda_i$$

$$|\psi(t)\rangle = e^{-i\overbrace{\frac{\lambda_i}{\hbar}}^{\omega_i} t} \qquad \lambda_i = \hbar\omega_i = E_i$$

$\hat{H}$ je operator celotne energije sistema. $\qquad \hat{H} = \hat{H}_{kin} + \hat{V}$

Stacionarna stanja $\qquad |\psi(t)\rangle = \underbrace{e^{-i\omega_i t}}_{\text{samo faza se spreminja!}} |i\rangle$

Osnovno stanje $|0\rangle$ $\qquad E_0 < E_1, E_2, E_3$

↑ vzbujena stanja $|1\rangle, |2\rangle, \ldots$

$$|\psi(t=0)\rangle = a\,|1\rangle + b\,|2\rangle$$

$$|\psi(t=0)\rangle = a\,e^{-i\omega_1 t}|1\rangle + b\,e^{-i\omega_2 t}|2\rangle$$

$$p_1(t) = |\langle 1|\psi\rangle|^2 = |a|^2 \qquad a = b = 1/\sqrt{2}$$

Ali je sistem v stanju $|s\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$?

$$p_s(t) = |\langle s|\psi\rangle|^2$$

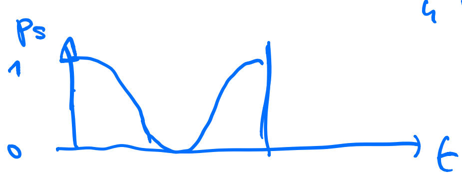
$$\langle s|\psi\rangle = \frac{1}{\sqrt{2}}\left(\langle 1| + \langle 2|\right)\left(a\,e^{-i\omega_1 t}|1\rangle + b\,e^{-i\omega_2 t}|2\rangle\right) = \frac{1}{\sqrt{2}}\left(a\,e^{-i\omega_1 t} + b\,e^{-i\omega_2 t}\right) = \frac{1}{2}\left(e^{-i\omega_1 t} + e^{-i\omega_2 t}\right)$$

$$p_s = \langle\psi|s\rangle\langle s|\psi\rangle = \frac{1}{4}\left(e^{-i\omega_1 t} + e^{-i\omega_2 t}\right)\left(e^{i\omega_1 t} + e^{i\omega_2 t}\right)$$

$$= \frac{1}{4}\left(1 + 1 + e^{i(\omega_1-\omega_2)t} + e^{-i(\omega_1-\omega_2)t}\right)$$

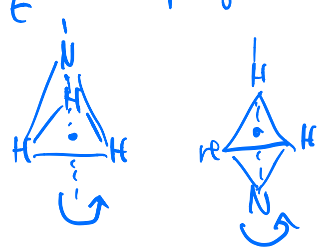
$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) = |s\rangle$

$$= \frac{1}{4}\left(2 + 2\cos(\omega_1-\omega_2)t\right) = \frac{1+\cos(\omega_1-\omega_2)t}{2}$$



utripanje

Amonijak NH_3



$|1\rangle \qquad |2\rangle$

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi\rangle$$

$|1\rangle, |2\rangle$
$$|\psi(t)\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \sum_{i=1}^{2} c_i(t)\,|i\rangle$$

$$\langle i| \Big/ \, i\hbar\frac{d}{dt}|\psi(t)\rangle = \langle i|\hat{H}|\psi\rangle$$

$$i\hbar\frac{d}{dt}\langle i|\psi(t)\rangle = \langle i|\hat{H}\hat{1}|\psi\rangle$$

$$= \langle i|\hat{H}|\sum_j |j\rangle\langle j|\psi\rangle$$

$$\sum_j |j\rangle\langle j| = 1$$

$$i\hbar\,\dot{c}_i(t) = \sum_j \underbrace{\langle i|\hat{H}|j\rangle}_{H_{ij}} c_j(t)$$

$i=1:$ $$i\hbar\,\dot{c}_1 = H_{11}c_1 + H_{12}c_2$$

$i=2:$ $$i\hbar\,\dot{c}_2 = H_{22}c_2 + H_{21}c_1$$

$$H_{12} = H_{21} = 0 \qquad i\hbar\,\dot{c}_1 = H_{11}c_1 \;\longrightarrow\; c_1 = e^{-i\frac{H_{11}}{\hbar}t}$$

$$i\hbar\,\dot{c}_2 = H_{22}c_2 \;\longrightarrow\; c_2 = e^{-i\frac{H_{21}}{\hbar}t}$$

$$H_{11} = H_{22} = E_0$$

$$H_{12} = H_{21}^* = -A$$ tuneliranje

$$i\hbar\,\dot{c}_1 = E_0 c_1 - A c_2$$

$$i\hbar\,\dot{c}_2 = E_0 c_2 - A c_1$$

$$i\hbar\frac{d}{dt}(c_1+c_2) = E_0(c_1+c_2) - A(c_1+c_2)$$

$$c_1+c_2 = a\,e^{-\frac{i}{\hbar}(E_0-A)t} \leftarrow$$

$$i\hbar\frac{d}{dt}(c_1-c_2) = E_0(c_1-c_2) + A(c_1-c_2)$$

$$c_1-c_2 = b\,e^{-i/\hbar(E_0+A)t} \leftarrow$$

$$2c_1 = a\,e^{-i/\hbar(E_0-A)t} + b\,e^{-i/\hbar(E_0+A)t}$$

$$2c_2 = a\,e^{-i/\hbar(E_0-A)t} - b\,e^{-i/\hbar(E_0+A)t}$$

$a\neq 0 \quad b=0 \quad c_1 = c_2 \quad E = E_0 - A$

$a=0 \quad b\neq 0 \quad c_1 = -c_2 \quad E = E_0 + A$

razcep nivoja