# KVANTNO RAČUNANJE IN KOMUNICIRANJE

## 1) TENZORSKI PRODUKT

$U \qquad V \qquad\qquad U\otimes V \qquad\qquad |u\rangle\otimes|v\rangle \in U\otimes V$

$|u\rangle \qquad |v\rangle$

$|0\rangle,|1\rangle \qquad |0\rangle,|1\rangle \qquad\qquad |0\rangle\otimes|0\rangle,\ |0\rangle\otimes|1\rangle,\ |1\rangle\otimes|0\rangle,\ |1\rangle\otimes|1\rangle$

$U\otimes V\otimes W \quad \to \quad |0\rangle\otimes|1\rangle\otimes|0\rangle, \ldots$

$\dim = 2^n$

$|\psi\rangle = \alpha|0\rangle+\beta|1\rangle \to$ 2 realna parametra

$\dim = 2^n \to 2\cdot 2^n - 1 - 1$
$= 2(2^n-1)$

$n=2 \qquad \dim = 2(2^2-1) = 6 \to$ kvantna prepletenost

$\hat{A} \quad \hat{B} \qquad \hat{A} \quad U \qquad \hat{B} \quad V \qquad\qquad \hat{A}\otimes\hat{B} \qquad U\otimes V$

$$(\hat{A}\otimes\hat{B})(|u\rangle\otimes|v\rangle) = (\hat{A}|u\rangle)\otimes(\hat{B}|v\rangle) \in U\otimes V$$

$\hat{A}\otimes 1\!\!1 \qquad 1\!\!1\otimes\hat{B}$

$\hat{A} = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{B} = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |u\rangle = |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |v\rangle = |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$(\sigma_x\otimes\sigma_z)(|1\rangle\otimes|0\rangle) = (\sigma_x|1\rangle)\otimes(\sigma_z|0\rangle) = |0\rangle\otimes(-|0\rangle) = -|0\rangle\otimes|0\rangle$$

$$\left\{\begin{aligned} &|u\rangle\otimes(|w\rangle+|z\rangle) = |u\rangle\otimes|w\rangle + |u\rangle\otimes|z\rangle \\ &(|u\rangle+|v\rangle)\otimes|w\rangle = |u\rangle\otimes|w\rangle + |v\rangle\otimes|w\rangle \\ &(k|u\rangle)\otimes|v\rangle = |u\rangle\otimes(k|v\rangle) = k\,|u\rangle\otimes|v\rangle \end{aligned}\right.$$

bilinearnost

$|\psi\rangle = \sum_i a_i\,|u_i\rangle\otimes|v_i\rangle \qquad |\varphi\rangle = \sum_i b_i\,|u_i\rangle\otimes|v_i\rangle$

$$\langle\psi|\varphi\rangle = \Big(\sum_i a_i^*\langle u_i|\otimes\langle v_i|\Big)\Big(\sum_j b_j\,|u_j\rangle\otimes|v_j\rangle\Big)$$

$$= \sum_{ij} a_i^* b_j\,\langle u_i|u_j\rangle\,\langle v_i|v_j\rangle$$

$$|u\rangle \otimes |v\rangle \equiv |u\rangle|v\rangle \equiv |u,v\rangle \equiv |uv\rangle$$

$$|0\rangle\otimes|0\rangle,\ |0\rangle\otimes|1\rangle, \dots \longrightarrow |00\rangle, |01\rangle, |10\rangle, |11\rangle$$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

kroneckerjev produkt $\qquad \sum_{i,j} |\alpha_{ij}|^2 = 1 \iff \langle\psi|\psi\rangle = 1$

$$\underset{(m\times n)}{\hat{A}} \otimes \underset{(p\times q)}{\hat{B}} = \begin{bmatrix} A_{11}\cdot B & A_{12}\cdot B & A_{13}\cdot B \dots & A_{1n}\cdot B \\ A_{21}\cdot B & & & \\ A_{31}\cdot B & & & \\ A_{m1}\cdot B & & & A_{mn}\cdot B \end{bmatrix} \Rightarrow (mp \times nq)$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} \qquad A\otimes B = \begin{pmatrix} 1\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} & 2\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} \\ 3\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} & 4 \dots \end{pmatrix} = \begin{pmatrix} 4 & 5 & 8 & 10 \\ 6 & 7 & 12 & 14 \\ 12 & 15 & 16 & 20 \\ 18 & 21 & 24 & 28 \end{pmatrix}$$

$$\sigma_x \otimes \sigma_z = \begin{bmatrix} \emptyset & 1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ 1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & \emptyset \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\sigma_z \otimes \sigma_x = \begin{bmatrix} 1(\sigma_x) & \emptyset \\ \emptyset & -1(\sigma_x) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad A\otimes B \neq B \otimes A$$

$$(A+B)\otimes C = A\otimes C + B\otimes C \qquad k(A\otimes B) = (kA)\otimes B = A\otimes(kB)$$

$$(A\otimes B)^\dagger = A^\dagger \otimes B^\dagger \qquad \text{POZOR!} \qquad (AB)^\dagger = B^\dagger A^\dagger$$

$$|\psi\rangle^{\otimes n} = \underbrace{|\psi\rangle \otimes |\psi\rangle \otimes \dots \otimes |\psi\rangle}_{n\times} \qquad |\psi\rangle^{\otimes 2} = |\psi\rangle\otimes|\psi\rangle$$

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad \sigma_x|\psi\rangle = |\psi\rangle \qquad |\psi\rangle^{\otimes 2} = \tfrac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

## 2) MERITVE V SISTEMIH VEČ DELCEV

$$p_i = |\langle i|\psi\rangle|^2 = \langle\psi|i\rangle\langle i|\psi\rangle = \langle\psi|\left(|i\rangle\langle i|\right)|\psi\rangle = \langle\psi|\hat{P}_i|\psi\rangle$$

$\alpha|0\rangle + \beta|1\rangle$ $\qquad$ $P_i = |i\rangle\langle i|$

$\mathbb{1} = |0\rangle\langle 0| + |1\rangle\langle 1|$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

Ali je prvi v stanju 0? $\quad \hat{P} = \left(|0\rangle\langle 0|\right)\otimes\mathbb{1} = \left(|0\rangle\langle 0|\right)\otimes\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

$$= |00\rangle\langle 00| + |01\rangle\langle 01|$$

$$p_{\text{prvi}=0} = |\alpha_{00}|^2 + |\alpha_{01}|^2$$

$$= \langle\psi|\left(|00\rangle\langle 00| + |01\rangle\langle 01|\right)|\psi\rangle$$

$$= \langle\psi|\left(\underbrace{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}\right) = \alpha_{00}^*\alpha_{00} + \alpha_{01}^*\alpha_{01} = |\alpha_{00}|^2 + |\alpha_{01}|^2$$

Če je prvi enak nič, ali je tudi drugi?

$$|\psi'\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle \qquad \langle\psi'|\psi'\rangle = |\alpha_{00}|^2 + |\alpha_{01}|^2$$

$$\downarrow$$

$$|\psi'\rangle = \boxed{\frac{1}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}}\left(\alpha_{00}|00\rangle + \alpha_{01}|01\rangle\right)$$

$$p_{\text{tudi drugi }0} = \frac{|\alpha_{00}|^2}{|\alpha_{00}|^2 + |\alpha_{01}|^2}$$

Ali je sistem v stanju 00? $\qquad p_{00} = |\alpha_{00}|^2$

$$p(A\cap B) = p(A|B)\,p(B) = p(B|A)\,p(A)$$

$$p_{00} = p_{\text{prvi je }0}\; p_{\text{tudi drugi je }0} = \left(|\alpha_{00}|^2 + |\alpha_{01}|^2\right)\left(\frac{|\alpha_{00}|^2}{|\alpha_{00}|^2 + |\alpha_{01}|^2}\right) = |\alpha_{00}|^2$$

## 3) KVANTNA PREPLETENOST

$$|\psi\rangle = 1/\sqrt{2}\left(|00\rangle + |11\rangle\right)$$

EPR paradox

Bellova stanja: $\qquad |\bar{0}\rangle = |1\rangle, \quad |\bar{1}\rangle = |0\rangle$

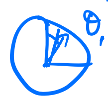
$$|\beta_{00}\rangle = 1/2\,(|00\rangle + |11\rangle)$$
$$|\beta_{10}\rangle = 1/\sqrt{2}\,(|00\rangle - |11\rangle)$$
$$|\beta_{01}\rangle = 1/\sqrt{2}\,(|01\rangle + |10\rangle)$$
$$|\beta_{11}\rangle = 1/\sqrt{2}\,(|01\rangle - |10\rangle)$$

$$|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}\left(|0y\rangle + (-1)^x |\bar{y}0\rangle\right)$$

$\langle\beta_{00}|\beta_{00}\rangle = 1 \quad \langle\beta_{11}|\beta_{00}\rangle = 0$ $\qquad$ $\beta_{ij}$ tvorijo bazo za prostor 2 kubitov

Maksimalno prepletena.

Separabilna. $(\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$



$\theta, \varphi \qquad \theta', \varphi' \quad \Rightarrow 4$ parametri

$(\frac{1}{\sqrt{2}}|0\rangle + |1\rangle)^{\otimes 2}$ $\quad \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) = \_\_\_ \otimes \_\_\_ \rightarrow$ povsem separabilno

4) IZREK O PREPOVEDI KLONIRANJA (Wooter, Zurek 1982)

Ali obstaja $\begin{cases} \hat{U}(|\psi\rangle \otimes |0\rangle) = |\psi\rangle \otimes |\psi\rangle \ ? \\ \hat{U}(|\varphi\rangle \otimes |0\rangle) = |\varphi\rangle \otimes |\varphi\rangle \end{cases}$ $\qquad \hat{U}^\dagger \hat{U} = 1 = \hat{U}\hat{U}^\dagger$

$$\langle\phi|\psi\rangle = (\langle\phi| \otimes \langle 0|)(|\psi\rangle \otimes |0\rangle) = (\langle\phi| \otimes \langle 0|)\,\hat{U}^\dagger \hat{U}\,(|\psi\rangle \otimes |0\rangle)$$
$$= (\langle\phi| \otimes \langle\phi|)(|\psi\rangle \otimes |\psi\rangle) = \langle\phi|\psi\rangle\langle\phi|\psi\rangle$$

$x = x^2$ $\nearrow$ $x = 0 \qquad \langle\phi|\psi\rangle = 0$

$\quad\;\;\searrow$ $x = 1 \qquad \langle\phi|\psi\rangle = 1 \qquad |\phi\rangle = |\psi\rangle$

Kopiranje je možno samo za pare ortogonalnih stanj.

$|0\rangle$ ali $|1\rangle \longrightarrow |00\rangle, \ |11\rangle \qquad \hat{U}$ (za par $|0\rangle$ in $|1\rangle$)

$\underbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}_{=|+\rangle}$ ali $\underbrace{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}_{=|-\rangle} \longrightarrow |++\rangle, \ |--\rangle \qquad \hat{U}$ (za par $|+\rangle$ in $|-\rangle$)

Izrek o prepovedi brisanja. $\qquad$ No-clone, no-delete theorem.

No-go theorems.

No-communication theorem.

$$|\beta_{00}\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

$\boxed{0}$: Izmerim v $\sigma_z$ bazi: $\rightarrow$ $|0\rangle$ ali $|1\rangle$

ali

$\boxed{1}$: Izmerim v $\sigma_x$ bazi: $\rightarrow$ $|+\rangle$ ali $|-\rangle$

$$|\beta_{00}\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle) = 1/\sqrt{2}\,(|++\rangle + |--\rangle)$$

$|0\rangle, |0\rangle, |0\rangle, |0\rangle, |0\rangle \longrightarrow 00000$

$|1\rangle, |1\rangle \ldots \longrightarrow 11111$ $\qquad \sigma_z \longrightarrow \boxed{0}$

$|+\rangle, |+\rangle, |+\rangle, 1/\sqrt{2}(|0\rangle+|1\rangle), \ldots$ $\quad 0011010100$ $\qquad \sigma_x \longrightarrow \boxed{1}$

$|-\rangle, |-\rangle$ $\quad 1/\sqrt{2}(|0\rangle-|1\rangle),$ $\quad 1101011110$

$$|v\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \langle v^\perp|v\rangle = \beta\alpha - \alpha\beta = 0$$

$$|v^\perp\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$$

$$|0\rangle \rightarrow a\,|v\rangle + b\,|v^\perp\rangle$$

$$|1\rangle \rightarrow -b\,|v\rangle + a\,|v^\perp\rangle \qquad a = b = 1/\sqrt{2}$$

$$|\beta_{00}\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

$$= 1/\sqrt{2}\,(a|v\rangle + b|v^\perp\rangle)(a|v\rangle + b|v^\perp\rangle)$$

$$+ 1/\sqrt{2}\,(-b|v\rangle + a|v^\perp\rangle)(-b|v\rangle + a|v^\perp\rangle)$$

$$= 1/\sqrt{2}\,(a^2 + b^2)\,|vv\rangle + 1/\sqrt{2}\,(a^2+b^2)\,|v^\perp v^\perp\rangle$$

$$|\beta_{00}\rangle = 1/\sqrt{2}\,(|vv\rangle + |v^\perp v^\perp\rangle)$$