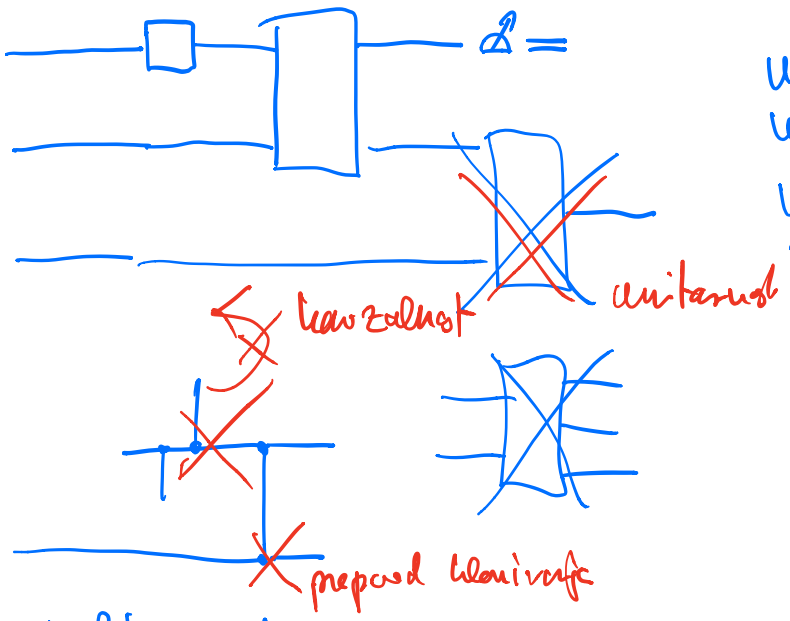
1) KVANTNA VEZJA

čas →



$U_x = 2$

$U_y = 2$

$U(x-y) = \mathbb{1}$

$x - y = 0$ $\quad x = y$

NAND

NOR

AND

OR

2) Evolucijska matrika

$-\boxed{\mathbb{1}}- \equiv ---$ $\qquad -\boxed{X}-$ $\qquad X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

NOT bit-flip

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$X(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle$

$X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$

PHASE-FLIP

$-\boxed{Z}-$ $\qquad -\boxed{Y}-$

$Z = \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

$-\boxed{H}-$ Hadamard $\qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\qquad H^2 = \mathbb{1}$

$$\begin{cases} H|0\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle) = |+\rangle \\ H|1\rangle = 1/\sqrt{2}\,(|0\rangle - |1\rangle) = |-\rangle \end{cases}$$

$$\begin{cases} H|+\rangle = |0\rangle \\ H|-\rangle = |1\rangle \end{cases}$$

S — Fazna vrata $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

T — $\pi/8$ vrata $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \rightarrow \begin{bmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{bmatrix}$

3) CNOT — H, S, T

kontrolni kubit

ciljni kubit

pogojna negacija

in		out	
k	c	k	c
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

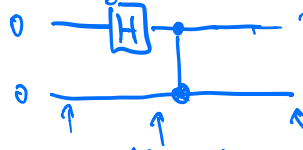
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$|A, B\rangle \xrightarrow{CNOT} |A, B \oplus A\rangle$

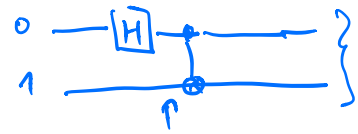
$a \oplus k = \begin{cases} 0 & \text{če } 00, 11 \\ 1 & \text{če } 01, 10 \end{cases}$

CNOT, H, S, T tvorijo univerzalni nabor kv. vrat

"Entangler"



$|00\rangle \quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \quad \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

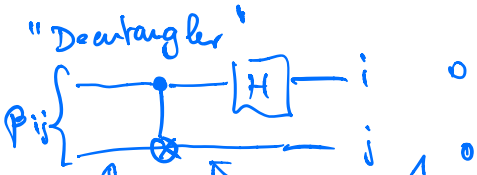
$$\beta_{ij} = \frac{1}{\sqrt{2}}\left(|0j\rangle + (-1)^i |1,\bar{j}\rangle\right)$$

$\hat{A}^{-1} = \hat{A}^\dagger$

$\hat{H}^\dagger = \hat{H}$

$CNOT^\dagger = CNOT$

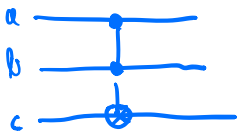
"Deentangler"



$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = |+\rangle \otimes |0\rangle$

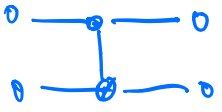
David Wineland 1995 > Nobelova 2012

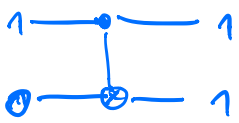
Toffoli

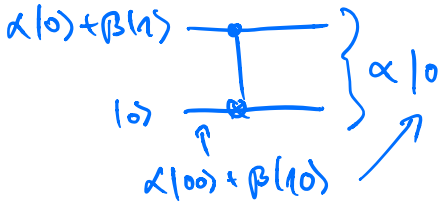


$|a,b,c\rangle \to |a,b,\ c\oplus ab\rangle$

4) Je kopiranje možno?





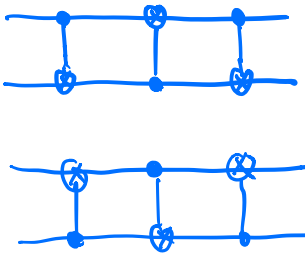


$\alpha|00\rangle + \beta|11\rangle$ ali $(\alpha|0\rangle+\beta|1\rangle)\otimes(\alpha|0\rangle+\beta|1\rangle)$

$\alpha^2|00\rangle + \alpha\beta|01\rangle + \beta\alpha|10\rangle + \beta^2|11\rangle$

$$\begin{cases} \alpha = \alpha^2 \\ \beta = \beta^2 \\ \alpha\beta = 0 \end{cases} \to \begin{array}{l} \alpha = 1\ \beta = 0 \\ \text{ali:} \\ \beta = 1\ \alpha = 0 \end{array}$$

5) SWAP



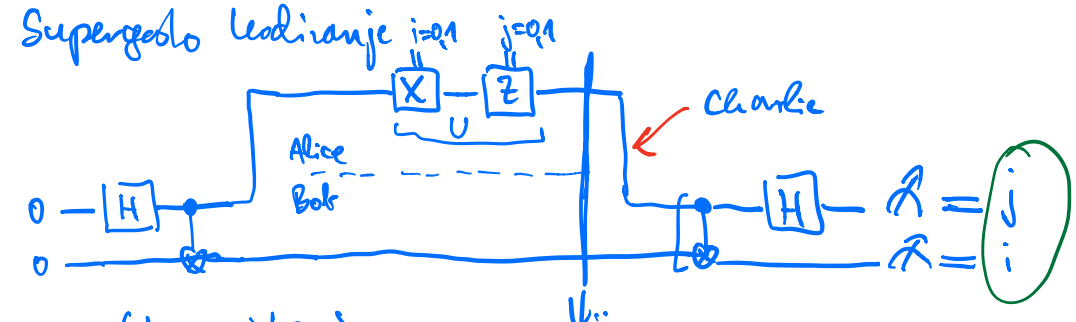
$|a,b\rangle \to |b,a\rangle$

$|a,b\rangle \Rightarrow |a,\ b\oplus a\rangle \Rightarrow |a\oplus(b\oplus a),\ b\oplus a\rangle$

$= |b,\ b\oplus a\rangle$

$\Rightarrow |b,\ b\oplus a\oplus b\rangle = |b,a\rangle$ ✓

6) Supergosto kodiranje



$|\psi_{ij}\rangle = (U_{ij}\otimes \mathbb{1})|\beta_{00}\rangle$

i	j	$\lvert\psi_{ij}\rangle$
0	0	$\lvert\beta_{00}\rangle = 1/\sqrt{2}(\lvert 00\rangle + \lvert 11\rangle)$
0	1	$1/\sqrt{2}(\lvert 00\rangle - \lvert 11\rangle) = \beta_{10}$
1	0	$1/\sqrt{2}(\lvert 10\rangle + \lvert 01\rangle) = \beta_{10}$
1	1	$1/\sqrt{2}(\lvert 10\rangle - \lvert 01\rangle) = \beta_{11}$

β_{ij}

$\beta_{ij} \rightarrow ij$