Diskretne strukture UNI
Vaje, 7. teden

1. Katere izmed spodnjih formul so enakovredne?

$$\begin{aligned} A &= \exists x(\forall y P(x,y) \Rightarrow \forall y R(x,y)), \\ B &= \exists x(\forall y P(y,x) \Rightarrow \forall y R(x,y)), \\ C &= \exists x(\forall y P(x,y) \Rightarrow \forall y R(y,x)). \end{aligned}$$

2. Pokaži, da sta formuli

$$F_1 = \neg\exists x((\neg R(x) \Rightarrow P(x)) \wedge (Q(x) \Rightarrow R(x)))$$

in

$$F_2 = \forall x(P(x) \Rightarrow Q(x)) \wedge \neg\exists y R(y)$$

enakovredni.

3. Dane so množice $A = \{1,2,3\}$, $B = \{2,3,4\}$ in $C = \{0,1,4,5\}$. Določi naslednje množice:
   (a) $C + (A \cup C)$,
   (b) $(B \setminus A) \cap C$,
   (c) $\mathcal{P}(A \cap B) \setminus B$,
   (d) $\mathcal{P}(A \cap C) + \mathcal{P}(B \cap C)$.

4. Na ravni elementov pokaži, da velja
   (a) $A \subseteq B \Rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$,
   (b) $A \subseteq B \Leftrightarrow A \cap B = A$.

5. Ali velja
   (a) $(A + B) \setminus A = B \setminus A$,
   (b) $(A + B) + (A + C) = A + (B + C)$,
   (c) $(A \setminus B) + (C \setminus B) = (A + C) \setminus B$,
   (d) $(A + C) \setminus (A + B) = (A \cap B) + C$,
   (e) $(A + C) \setminus (A + B) = (A \cap B) + C$ pod pogojem $C \subseteq A \cap B$,
   (f) $(A + C) \setminus (A + B) \subseteq (A \cap B) + C$,
   (g) $(A + B) \setminus C \subseteq (B \setminus (A + C)) \cup (A \setminus (B \cup C))$,
   (h) $(A + B) \setminus C = (B \setminus (A + C)) \cup (A \setminus (B \cup C))$, če sta A in B disjunktni,
   (i) $(A + B) \setminus C = (A \cup C) + (A \cup B)$,
   (j) $(A \cap C) + (B \cap C) = C \setminus (A \cap B)$,
   (k) $(A \cap C) + (B \cap C) \subseteq C \setminus (A \cap B)$,
   (l) $(A \cap C) + (B \cap C) = C \setminus (A \cap B)$, če je $C \subseteq A \cup B$,
   (m) $(A \setminus C) + B = (A + B) \setminus C$,
   (n) $(A \setminus C) + B \subseteq (A + B) \setminus C$,
   (o) $(A \setminus C) + B = (A + B) \setminus C$, če je $C \subseteq A \setminus B$?

6. Pokaži, da množice $B \cap C$, $(B + C) \cap A$ in $(A + C) \setminus B$ predstavljajo razbitje za množico $A \cup C$.